Some Mathematical Models on Growth Problems and Nonlinear Elliptic Equations

Ireneo Peral

Departamento de Matemáticas, UAM, Madrid, Spain

ireneo.peral@uam.es

Abstract

Epitaxial growth is characterized by the deposition of new material on existing layers of the same material under high vacuum conditions.

We will assume that the height function obeys a gradient flow equation with a forcing term

$$u_t = \sqrt{1+(\nabla u)^2}\left(-\frac{\delta \mathcal{J}(u)}{\delta u} + f(x,t)\right). \tag{1}$$

The functional $\mathcal{J}$ denotes a potential which describes the microscopic properties of the interface and, at the macroscopic scale, it is assumed that it can be expressed as a function of the surface mean curvature only [3]

$$\mathcal{J}(u) = \int_\Omega F(H)\sqrt{1+(\nabla u)^2}\, dx, \tag{2}$$

where the presence of the square root terms models growth along the normal to the surface, H denotes the mean curvature and F is an unknown function of H.

We focalize the study on the stationary problem where some borderline cases appear by studying problem of existence of solution. The central model in this work is given by the following elliptic equation,

$$\begin{cases} \Delta^2 u = \det\left(D^2 u\right) + \lambda f & x \in \Omega \subset \mathbb{R}^2 \\ \text{boundary conditions.} \end{cases}$$

There are many related problems, starting with the evolution problem associated to this elliptic problem.

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