Calc. Var. (2011) 40:291–292
DOI 10.1007/s00526-010-0368-5

Calculus of Variations

ERRATUM

Erratum to: Fast and heteroclinic solutions for a second order ODE related to Fisher-Kolmogorov's equation

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Published online: 19 October 2010
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Erratum to: Calc. Var. (2004) 21:319–334
DOI 10.1007/s00526-004-0264-y

The proof of Proposition 11 in [1] has a gap. The use of differential inequalities in line 7 of the statement in the beginning of page 333, is not correct. The aim of this note is to present a correct proof of this result.

The question is to prove that if the equation (1) has a non fast heteroclinic solution for some $c_1 > 2\sqrt{f'(0)}$, then $c_1 > c^*$, where $c^* := \inf\{c \in \mathbb{R} : (1)\ \text{ has an heteroclinic solution}\}$. So, we only have to prove there exists an heteroclinic solution for values below c_1.

By the claim in page 332 in [1] it is sufficient to find a solution of (19) for some $c < c_1$ with $y(0) = 0$ and $y(u) > 0,\ u \in]0, 1]$.

Let y_1 be the solution of (19) with $c = c_1$ corresponding to the non fast heteroclinic solution of (1). Then y_1 satisfies $y_1(0) = y_1(1) = 0$.

Consider $2\sqrt{f'(0)} < c_2 < c_1$ as in the beginning of page 333 in [1] and let y_2 be the solution of (19) with $c = c_2$ that satisfies $y_2(0) = 0$ and $y_2(u) > y_1(u)$ for small values

Communicated by P. Rabinowitz.

The online version of the original article can be found under doi:10.1007/s00526-004-0264-y.

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of u. Now we have to change the argument since differential inequalities do not prove that $y_2(u) > y_1(u)$, $u \in]0, 1]$. However we can find a value of c between c_2 and c_1 for which (19) has a solution with $y(0) = 0$ and $y(u) > 0$, $u \in]0, 1]$. Indeed:

Take $b_n \to 0$ and $\tilde{c}_n \to c_1$ two sequences satisfying $b_n > 0$ and $c_2 < \tilde{c}_n < c_1$ and denote $\tilde{y}_n$ the solution of (19) with $c = \tilde{c}_n$ and $\tilde{y}_n(1) = b_n$.

Since $\tilde{c}_n < c_1$ then $\tilde{y}_n(u) > y_1(u)$ for $u \in]0, 1]$. Indeed, let $\bar{u_n} := \inf\{u_n > 0 : y_1(u) < \tilde{y}_n(u),\ u \in (u_n, 1)\}$. By a contradiction argument, if $\bar{u_n} > 0$ then $y_1(\bar{u_n}) = \tilde{y}_n(\bar{u_n})$ and $y_1'(\bar{u_n}) \leq \tilde{y}_n'(\bar{u_n})$ and this is impossible because, by (19) $\tilde{y}_n'(\bar{u_n}) - y_1'(\bar{u_n}) = 2(\tilde{c}_n - c_1)\sqrt{\tilde{y}_n(\bar{u_n})}$, and this is strictly negative.

By standard arguments $\tilde{y}_n \to \tilde{y}_0$ uniformly in $[0, 1]$ and $\tilde{y}_0$ is a solution of (19) that satisfies $\tilde{y}_0(1) = 0$. Let us observe that this last initial value problem has uniqueness to the left, since the right hand term of differential equation (19) is increasing. So $\tilde{y}_0 \equiv y_1$ and if we fix $\bar{u}$ sufficiently near 0 where y_2 is defined, for n large enough $y_1(\bar{u}) < \tilde{y}_n(\bar{u}) < y_2(\bar{u})$. Now since $c_2 < \tilde{c}_n$ an argument as the previous one shows that this last inequality is true for $u \in (0, \bar{u})$ and so $\tilde{y}_n(0) = 0$.

Acknowledgement The authors wish to express their gratitude to Cristina Marcelli for having pointed out the gap in the proof.