The group structure of non-Abelian NS-NS transformations

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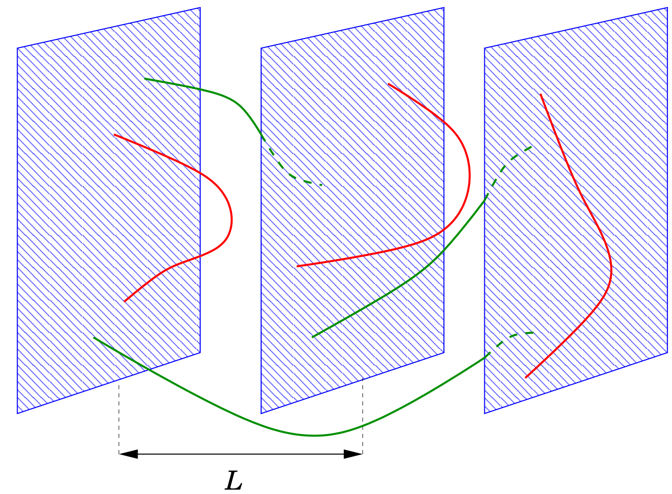
References: JHEP 05 (2010) 035, arXiv:1003.5317

Motivation

A lot is known about the dynamics of multiple coinciding D-branes:



strings between same brane: $m \sim 0$

strings between different branes: $m \sim L$

As $L \to 0$:

$\Longrightarrow N + N(N-1) = N^2$ degrees of freedom

$\Longrightarrow U(1)^N \to U(N)$ gauge enhancement

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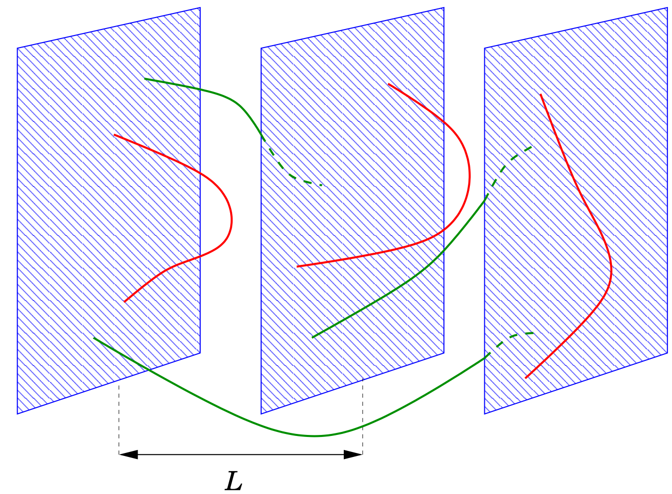

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Different degrees of freedom and different symmetries

$\Longrightarrow$ different dynamics

$\Longrightarrow$ different worldvolume action

$$\mathcal{L}_{D1} \sim \mathrm{STr}\Big\{ C_{\mu\nu} D_a X^\mu D_b X^\nu + i[X^\mu, X^\nu] C_{\mu\nu\rho\lambda} D_a X^\rho D_b X^\lambda + \ldots \Big\}$$

[Taylor, van Raamsdonk][Myers]

Many applications in modern string theory

Question: Is the constructed action (dynamics) gauge invariant?

- $U(N)$ invariance is straight forward.

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- Background gauge invariance is less obvious.
  - R-R and massive gauge transformations $\delta C = \partial \Lambda e^B + m \Sigma e^B$:
    Non-Abelian generalization of known (Abelian) results

    [Green, Hull, Townsend, 96; Ciocarli, 2001] [Adam, Gheeardyn, B.J.,Lozano, 2003] [Adam, 2005]

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  - NS-NS transformation: [Adam, Illán, B.J., 2005]

    $$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]} \qquad \delta V_a = -\Sigma_\mu D_a X^\mu \qquad \delta X^\mu = i\Sigma_\rho [X^\rho, X^\mu]$$

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Aim: Explore the group structure of the non-Abelian NS-NS transformations

Outlook

1. Derivation of transformations rule via T-duality

2. Simple case: Pure gauge NS-NS transformations

3. Full NS-NS algebra and Jacobi identity

4. Conclusions

1. Derivation of the transformation rules via T-duality

T-duality in worlvolume direction $\underline{x}$:

$$\begin{array}{ccc} \text{D}p\text{-brane} & \longrightarrow & \text{D}(p-1)\text{-brane} \\ (\hat{V}_{\hat{a}}, Y^i) & \longrightarrow & (V_a, X^{\hat{i}}) \end{array}$$

- Matching of degrees of freedom: [Bergshoeff, de Roo]

$$\hat{V}_a \longrightarrow V_a, \qquad Y^i \longrightarrow X^i,$$

$$\hat{V}_x \longrightarrow X^{\underline{x}}, \qquad Y^{\underline{x}} = \sigma^x.$$

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- Field strengths, Abelian case:

$$\partial_a Y^i \longrightarrow \partial_a X^i, \qquad \partial_x Y^i = 0, \qquad \partial_{\hat{a}} Y^{\underline{x}} = \delta_{\hat{a}}^{\underline{x}},$$
$$\hat{F}_{ax} \longrightarrow \partial_a X^{\underline{x}}, \qquad \hat{F}_{ab} \longrightarrow F_{ab}.$$

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- Field strengths, non-Abelian case: $(D_a Y^\mu = \partial_a Y^\mu + i[V_a, Y^\mu])$ [Myers]

$$\hat{D}_a Y^i \longrightarrow D_a X^i, \qquad \hat{D}_x Y^i = i[V_x, Y^i] \longrightarrow i[X^{\underline{x}}, X^i], \qquad \partial_{\hat{a}} Y^{\underline{x}} = \delta_{\hat{a}}^{\underline{x}},$$
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- Role played by fields in Dp-brane action is the same as role of fields in D$(p-1)$-brane action.

  Fields in both actions transform in the same way under:

  – Worldvolume general coordinate transformations ζ^a

  – $U(1)$ or $U(N)$ transformations χ

  – NS-NS transformations Σ

  – Target space general coordinate transformations

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$\Longrightarrow$ Derive transformation rules from T-duality between Dp-brane actions

T-duality on gauge transformations: Abelian case

T-duality in worlvolume direction $\underline{x}$:

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T-dual variation of BI vector: $\hat{V}_a \longrightarrow V_a$

$$\Longrightarrow \delta\hat{V}_a = \hat{\zeta}^b\partial_b\hat{V}_a + \partial_a\hat{\zeta}^b\hat{V}_b + \partial_a\hat{\zeta}^x\hat{V}_x + \partial_a\hat{\chi} - \Sigma_{\hat{\mu}}\partial_a Y^{\hat{\mu}}$$

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T-duality rules: $\hat{\zeta}^x \rightarrow \Sigma_{\underline{x}}$ $\qquad (g_{\mu\underline{x}} \rightarrow B_{\mu\underline{x}})$

$$\tilde{\chi} \rightarrow \chi - \Sigma_{\underline{x}} X^{\underline{x}}$$

$$\longrightarrow \zeta^b\partial_b V_a \;+\; \partial_a\zeta^b V_b \;+\; \partial_a\Sigma_{\underline{x}} X_{\underline{x}} \;+\; \partial_a\tilde{\chi} \;-\; \Sigma_\mu\partial_a X^\mu$$

$$= \zeta^b\partial_b V_a \;+\; \partial_a\zeta^b V_b \;+\; \partial_a\chi \;-\; \Sigma_{\hat{\mu}}\partial_a X^{\hat{\mu}}$$

$$= \delta V_a$$

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$$\longrightarrow \; \zeta^b \partial_b X^{\underline{x}} \;-\; \xi^{\underline{x}}$$

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$\longrightarrow$ $X^{\underline{x}}$ behaves as a scalar in worldvolume and coordinate in target space.

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Worldvolume transformations:

$$\boxed{\begin{aligned} \delta V_a &= \zeta^b \partial_b V_a + \partial_a \zeta^b V_b + \partial_a \chi - \Sigma_{\hat{\mu}} \partial_a X^{\hat{\mu}} \\ \delta X^{\hat{\mu}} &= \zeta^b \partial_b X^{\hat{\mu}} - \xi^{\hat{\mu}} \end{aligned}}$$

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$\longrightarrow$ V_a behaves as a vector in worldvolume, $U(N)$ Yang-Mills vector and shift under NS-NS transf .

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[Adam, Illán, B.J.]

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&\longrightarrow \zeta^b\partial_b\hat{V}_{\underline{x}} + i[X^{\underline{x}},\chi] - i\Sigma_\mu[X^{\underline{x}},X^\mu] - \xi^{\underline{x}} \\
&= \delta X^{\underline{x}}
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[Adam, Illán, B.J.]

$\longrightarrow$ $X^{\underline{x}}$ behaves as a scalar in worldvolume, adjoint scalar under $U(N)$, coordinate in target space and has extra "dielectric" Σ variation:

$$
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\delta V_a &= \zeta^b\partial_b V_a + \partial_a\zeta^b V_b + D_a\chi - \Sigma_{\hat{\mu}}D_aX^{\hat{\mu}} \\
\delta X^{\hat{\mu}} &= \zeta^b\partial_b X^{\hat{\mu}} + i[X^\mu,\chi] - i\Sigma_\rho[X^\mu,X^\rho] - \xi^{\hat{\mu}}
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Non-Abelian NS-NS transformations

Basic transformation rules:

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}, \qquad \delta V_a = -\Sigma_\mu D_a X^\mu, \qquad \delta X^\mu = i\Sigma_\rho[X^\rho, X^\mu]$$

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Composite transformation rules:

$$\delta\Phi(X) = i\,\Sigma_\mu[X^\mu, \Phi(X)]$$

$$\delta D_a X^\mu = i\Sigma_\rho[X^\rho, D_a X^\mu] + 2i[X^\mu, X^\lambda]\partial_{[\lambda}\Sigma_{\nu]}D_a X^\nu$$

$$\delta[X^\mu, X^\nu] = i\Sigma_\rho[X^\rho, [X^\mu, X^\nu]] + 2i[X^\mu, X^\rho]\partial_{[\rho}\Sigma_{\lambda]}[X^\lambda, X^\nu]$$

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⟶ with these transformation rules one can prove gauge invariance of Dp-brane action (at least Chern-Simons part) [Adam, Illán, B.J.]

⟶ what about the symmetry algebra, as $[\delta_{\Sigma_1}, \delta_{\Sigma_2}] \neq 0$?

2. Simple case: Pure gauge NS-NS transformations

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}, \qquad \delta X^\mu = \ i\Sigma_\rho[X^\rho, X^\mu], \qquad \delta V_a = \ -\Sigma_\mu D_a X^\mu$$

Transformations that leave $B_{\mu\nu}$ invariant, but act on X^μ and V_a: $\Sigma_\mu = \partial_\mu\chi$:

$$\delta B_{\mu\nu} = 0, \qquad \delta X^\mu = \ i\partial_\rho\chi[X^\rho, X^\mu], \qquad \delta V_a = \ -\partial_\mu\chi D_a X^\mu$$

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Properties of non-Abelian calculus:

$$[\chi(X), X^\mu] = \partial_\rho\chi(X)[X^\rho, X^\mu] \qquad D_a\chi(X) \ = \ \partial_\mu\chi(X) D_a X^\mu$$

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$$\Longrightarrow \qquad \delta B_{\mu\nu} = 0 \qquad \delta X^\mu = i[\chi, X^\mu] \qquad \delta V_a = -D_a\chi$$

$$\delta[X^\mu, X^\nu] = i[\chi, [X^\mu, X^\nu]], \qquad \delta D_a X^\mu = i[\chi, D_a X^\mu]$$

$\longrightarrow$ pure gauge part of NS-NS is $U(N)$ transformation

- Intertwining between NS-NS and $U(N)$
- What about the rest?

3. Full NS-NS algebra

Calucate commutators

$$[U(N), U(N)], \qquad [U(N), \text{NS-NS}] \qquad [\text{NS-NS}, \text{NS-NS}]$$

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- $[\delta_{\chi_1}, \delta_{\chi_2}] = \delta_{\chi_3}$ with $\chi_3 = i[\chi_1, \chi_2]$
- $$[\delta_\Sigma, \delta_\chi] X^\mu = -\Sigma_\rho [X^\rho, [\chi, X^\mu]] + [\chi, X^\rho][\Sigma_\rho, X^\mu] - [\chi, \Sigma_\rho][X^\rho, X^\mu] + [\chi, \Sigma_\rho [X^\rho, X^\mu]]$$

$$[\delta_\Sigma, \delta_\chi] V_a = i\partial_\mu \Sigma_\rho D_a X^\mu [X^\rho, \chi] + i\Sigma_\rho [D_a X^\rho, \chi] + i\Sigma_\rho [X^\rho, D_a \chi]$$

3. Full NS-NS algebra

Calucate commutators

$[U(N), U(N)]$, $[U(N), \text{NS-NS}]$ $[\text{NS-NS}, \text{NS-NS}]$

- $[\delta_{\chi_1}, \delta_{\chi_2}] = \delta_{\chi_3}$ with $\chi_3 = i[\chi_1, \chi_2]$

- $$\begin{aligned} [\delta_\Sigma, \delta_\chi] X^\mu &= -\Sigma_\rho [X^\rho, [\chi, X^\mu]] + [\chi, X^\rho][\Sigma_\rho, X^\mu] \\ &\quad - [\chi, \Sigma_\rho][X^\rho, X^\mu] + [\chi, \Sigma_\rho [X^\rho, X^\mu]] \\ &= i[\tilde{\chi}, X^\mu] \end{aligned}$$

$$\begin{aligned} [\delta_\Sigma, \delta_\chi] V_a &= i\partial_\mu \Sigma_\rho D_a X^\mu [X^\rho, \chi] + i\Sigma_\rho [D_a X^\rho, \chi] + i\Sigma_\rho [X^\rho, D_a \chi] \\ &= D_a \tilde{\chi} \end{aligned}$$

$[\delta_\Sigma, \delta_\chi] = \delta_{\tilde{\chi}}$ with $\tilde{\chi} = i\Sigma_\rho [X^\rho, \chi]$

- $[\delta_{\Sigma^{(1)}}, \delta_{\Sigma^{(2)}}]X^\mu \;=\; i\Big[i\Sigma^{(1)}_\lambda\Sigma^{(2)}_\rho[X^\lambda, X^\rho], X^\mu\Big] \;=\; i[\bar{\chi}, X^\mu]$

$$[\delta_{\Sigma^{(1)}}, \delta_{\Sigma^{(2)}}]V_a \;=\; D_a\Big(i\Sigma^{(1)}_\lambda\Sigma^{(2)}_\rho[X^\lambda, X^\rho]\Big) \;=\; D_a\bar{\chi}$$

with $\bar{\chi} = i\Sigma^{(1)}_\lambda\Sigma^{(2)}_\rho[X^\lambda, X^\rho]$

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with $\bar{\chi} = i\Sigma^{(1)}_\lambda\Sigma^{(2)}_\rho[X^\lambda, X^\rho]$

Full $U(N)$-NS-NS algebra:

$$[\delta_{\chi_1}, \delta_{\chi_2}] = \delta_{\chi_3} \qquad \text{with} \quad \chi_3 = i[\chi_1, \chi_2],$$

$$[\delta_\Sigma, \delta_\chi] = \delta_{\tilde{\chi}} \qquad \text{with} \quad \tilde{\chi} = i\Sigma_\rho[X^\rho, \chi],$$

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$\longrightarrow$ All commutators result in $U(N)$ transformation

Jacobi identities

$$[\delta_{\chi_1},[\delta_{\chi_2},\delta_{\chi_3}]]Z \;+\; [\delta_{\chi_2},[\delta_{\chi_3},\delta_{\chi_1}]]Z \;+\; [\delta_{\chi_3},[\delta_{\chi_1},\delta_{\chi_2}]]Z \;=\; i[\chi_0,Z],$$

$$[\delta_{\Sigma},[\delta_{\chi_1},\delta_{\chi_2}]]Z \;+\; [\delta_{\chi_2},[\delta_{\Sigma},\delta_{\chi_1}]]Z \;+\; [\delta_{\chi_1},[\delta_{\chi_2},\delta_{\Sigma}]]Z \;=\; i[\tilde{\chi},Z],$$

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with

$$\chi_0 \;=\; [\chi_1,[\chi_2,\chi_3]] \;+\; [\chi_2,[\chi_3,\chi_1]] \;+\; [\chi_3,[\chi_1,\chi_2]] \;=\; 0,$$

$$\tilde{\chi} \;=\; \Sigma_\rho\Big([[\chi_1,\chi_2],X^\rho] + [[X^\rho,\chi_1],\chi_2] + [[\chi_2,X^\rho],\chi_{]}\Big) \;=\; 0,$$

$$\bar{\chi} \;=\; -\Sigma^{(1)}_\lambda\Sigma^{(2)}_\rho\Big([X^\lambda,[X^\rho,\chi]] + [\chi,[X^\lambda,X^\rho]] + [X^\rho,[\chi,X^\lambda]]\Big) \;=\; 0,$$

$$\hat{\chi} \;=\; -\Sigma^{(1)}_\nu\Sigma^{(2)}_\lambda\Sigma^{(3)}_\rho\Big([X^\nu,[X^\lambda,X^\rho]] + [X^\rho,[X^\nu,X^\lambda]] + [X^\lambda,[X^\rho,X^\nu]]\Big) \;=\; 0$$

$\longrightarrow$ NS-NS Jacobi identities satisfied thanks to $U(N)$ Jacobi identities

4. Conclusions

- NS-NS transformations act non-trivially on worldvolume fields

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]}, \qquad \delta X^{\mu} = i\Sigma_{\rho}[X^{\rho}, X^{\mu}], \qquad \delta V_a = -\Sigma_{\mu} D_a X^{\mu}$$

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- $U(N)$ modifies NS-NS but in mild way:

$$\frac{\text{NS-NS}}{U(N)} \ = \ U(1)$$

$\longrightarrow U(N)$ "non-Abelianises" NS-NS

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Thanks!