Probes in fluxbranes and supersymmetry breaking through Hodge-duality

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Modest Question:

Can Hodge-duality break supersymmetry?

Answer:

Yes!

Reason:

Analogous to supersymmetry breaking in T-duality and dimensional reduction.

[Bakas, 1994]

[Bergshoeff, Kallosh, Ortín, 1994]

Overview

1. Reduction of M2-brane to D2-brane
    - Supergravity solutions
    - World volume actions

2. Reductions of Minkowski space to (supersymmetric) fluxbranes

3. Supersymmetry breaking through Hodge-duality

4. General considerations

1. Reduction M2 $\rightarrow$ D2

Supergravity solutions

The M2-brane solution of $D = 11$ supergravity

$$
\begin{aligned}
d\hat{s}^2 &= H^{-\frac{2}{3}}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + H^{\frac{1}{3}}\, (dx_i^2 + dz^2) \\
\hat{C}_{\mu\nu\rho} &= \epsilon_{\mu\nu\rho}\, H^{-1}
\end{aligned}
$$

reduces, after reduction over z to the D2-brane solution of Type IIA

$$
\begin{aligned}
ds^2 &= H^{-\frac{1}{3}}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + H^{\frac{1}{2}}\, dx_i^2 \\
e^{-2\phi} &= H^{-\frac{1}{2}} \\
C_{\mu\nu\rho} &= \epsilon_{\mu\nu\rho}\, H^{-1}
\end{aligned}
$$

with

$$
\begin{aligned}
H &= H(x^i), \qquad && \partial_i \partial^i H = 0 \\
\mu &= 0, 1, 2, \qquad && i = 1, ..., 8
\end{aligned}
$$

[Townsend, 1995]

World volume theories

M2: $D=3$, $N=8$ theory with 8 embedding scalars ϕ^i and Z and one fermion ψ:

$$\mathcal{L}_{M2} = -\tfrac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - \tfrac{1}{2}\partial_\mu Z\partial^\mu Z \ + \ \bar{\psi}\partial\!\!\!/\psi$$

$$\delta\phi^i = \bar{\epsilon}\,\Gamma^i\,\psi\,, \qquad \delta Z = -\mathrm{i}\,\bar{\epsilon}\,\psi\,,$$

$$\delta\psi = -\tfrac{1}{2}\Gamma^i\,\partial\!\!\!/\phi^i\,\epsilon \ - \ \tfrac{1}{2}\,\mathrm{i}\,\partial\!\!\!/Z\,\epsilon$$

D2: $D=3$, $N=8$ theory with 7 embedding scalars ϕ^i, one Born-Infeld vector A_μ and one fermion ψ :

$$\mathcal{L}_{D2} = -\tfrac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i + \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ \bar{\psi}\partial\!\!\!/\psi$$

$$\delta\phi^i = \bar{\epsilon}\,\Gamma^i\,\psi\,, \qquad \delta A_\mu = \bar{\epsilon}\,\tau_\mu\psi\,,$$

$$\delta\psi = -\tfrac{1}{2}\Gamma^i\,\partial\!\!\!/\phi^i\,\epsilon \ - \ \tfrac{1}{4}\tau^{\mu\nu}F_{\mu\nu}\epsilon$$

with Γ^i gamma-matrices of $SO(7)$

τ^μ gamma matrices of $SO(2,1)$

ψ bi-spinors of $SO(7)\times SO(2,1)$

$\mathcal{L}_{M2}$ and $\mathcal{L}_{D2}$ are related via Hodge-duality!

Consider $\Sigma_\mu = \partial_\mu Z$ an independent variable and impose Bianchi identity

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i - \tfrac{1}{2}\Sigma_\mu\Sigma^\mu + \bar{\psi}\partial\!\!\!/\psi \\ &\quad -\epsilon_{\mu\nu\rho}A_\mu\partial_\nu\Sigma_\rho\end{aligned}$$

- Eqn of motion of A_μ yields

$$\epsilon_{\mu\nu\rho}\partial_\nu\Sigma_\rho = 0 \qquad \Longrightarrow \qquad \Sigma_\mu = \partial_\mu Z$$

  Substitute $\longrightarrow$ $\mathcal{L}_{M2}$

- Eqn of motion of Σ_μ yields duality condition

$$\partial_\mu Z = -\tfrac{1}{2}\epsilon_{\mu\nu\kappa}F^{\nu\kappa}$$

  Substitute $\longrightarrow$ $\mathcal{L}_{D2}$

[Townsend, 1995]

2. Reduction to fluxbranes

$D = 11$ Minkowski space

$$ds^2 = -dt^2 + dx_m^2 + dr^2 + r^2 d\theta^2 + dz^2$$

with

$$z = z + 2\pi n_1 R, \qquad \theta = \theta + 2\pi n_2 + 2\pi n_1 RB$$

Define $\varphi = \theta - Bz$:

$$\begin{aligned} ds^2 &= -dt^2 + dx_m^2 + dr^2 + r^2 d\varphi^2 \\ &\quad + 2Br^2 d\varphi dz + (1 + B^2 r^2) dz^2 \end{aligned}$$

Dimensional reduction over z yields a (dilatonic) Melvin Universe

$$ds = \Lambda^{\frac{1}{2}}(-dt^2 + dx_m^2 + dr^2) + \Lambda^{-\frac{1}{2}} r^2 d\varphi^2$$

$$e^{-2\phi} = \Lambda^{\frac{3}{2}} \qquad F_{r\varphi} = \Lambda^{-2} Br$$

$$\Lambda(r) = 1 + B^2 r^2$$

[Melvin, 1964]

[Gibbons, Maeda, 1988]

Supersymmetric fluxbranes

Consider Killing vector $\xi = \partial_z + B_i{}^j x^i \partial_j$

(∂_z: translation, $B_i{}^j x^i \partial_j$: $SO(8)$ rotation)

Coordenates adapted to ξ: $\vec{y} = e^{-zB}\vec{x}$

$$d\hat{s}^2 = -dt^2 + d\vec{y}^2 + \Lambda dz^2 - 2B_{ij}y^i dy^j dz$$

with $\Lambda = 1 - y^i B_{ij} B_{jk} y^k$

$\xi \, \vec{y} = 0$

Reduction over z yield

$$ds^2 = \Lambda^{\frac{1}{2}} \left[-dt^2 + \left(\delta_{ij} + \Lambda^{-1} B_{ik} y^k B_{jl} y^l \right) dy^i dy^j \right]$$

$$\phi = \tfrac{3}{4} \ln \Lambda, \qquad C_i = \Lambda^{-1} B_{ij} y^j \; .$$

Amount of preserved supersymmetry depends on $SO(8)$ rotation B_{ij}!

[Figueroa-O'Farrill, Simón, 2001]

Amount of supersymmetry after reduction =
♯ of spinor components ϵ independent of compact direction

$$\mathcal{L}_\xi\, \epsilon = \xi^A \nabla_A \epsilon \;+\; \tfrac{1}{4}\partial_A \xi_B \gamma^{AB}\epsilon \;=\; 0,$$

$$\iff \quad \not{B}\epsilon \;\equiv B_{ij}\gamma^{ij}\epsilon \;= 0$$

$\iff$ ♯ of spinor components ϵ left invariant by B

$$\iff \quad B_{ij} \in \text{ isotropy group of } \epsilon$$

$B \in$	$\mathbb{1}$	$\mathfrak{sp}(1)$	$\mathfrak{su}(3)$	$\mathfrak{su}(4)$	$\mathfrak{so}(7)$
Q	32	16	8	4	0
IIA	M_{10}	$\mathcal{F}5$	$\mathcal{F}3$	$\mathcal{F}1$	“Melvin”

[Gutperle, Strominger, 2001]

[Uranga, 2001]

[Russo, Tseytlin, 2001]

[Figueroa-O’Farrill, Simón, 2001]

3. Supersymmetry breaking through Hodge-duality

$D = 3$ *theory with 8 scalars and one fermion and*
$N = 8$ *supersymmetry*
(M2-probe in D=11 Minkowski space)

$\downarrow$ Reduction over $\xi = \partial_z + B_i{}^j x^i \partial_j$

$\downarrow$ Hodge-dualisation of scalar in vector:

$$\partial_\mu Z \sim \epsilon_{\mu\nu\rho} F^{\nu\rho}(A)$$

$D = 3$ *theory with 7 scalars, one vector and one fermion and* $N = \not{B}\epsilon$ *supersymmetry*
(D2-probe in D=10 fluxbrane background)

NB: *At the level of the world volume theory, supersymmetry is broken through Hodge-duality!!*

$D=3$ theory with 8 scalars and one fermion and $N=8$ supersymmetry:

$$\mathcal{L}_{M2} = -\tfrac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i \;-\; \tfrac{1}{2}\partial_\mu Z\partial^\mu Z \;-\; \bar\psi\partial\!\!\!/\psi$$

invariant under

$$\delta\phi^i = \bar\epsilon\,\Gamma^i\,\psi \qquad\qquad \delta Z = -\,\mathrm{i}\,\bar\epsilon\,\psi$$

$$\delta\psi = -\tfrac{1}{2}\Gamma^i\,\partial\!\!\!/\phi^i\,\epsilon \;-\; \tfrac{1}{2}\,\mathrm{i}\,\partial\!\!\!/ Z\,\epsilon$$

$$\Rightarrow \begin{cases} [\delta_1,\delta_2] = -(\bar\epsilon_2\tau^\mu\epsilon_1)\partial_\mu \\ \delta\mathcal{L}_{M2} = 0 \end{cases}$$

Go to coordenates adapted to Killing vector $\xi = \partial_z + B_i{}^j x^i\partial_j$:

$$\chi^i = (e^{-ZB})^i{}_j\;\phi^j, \qquad\qquad \theta = e^{-\frac{1}{4}Z B\!\!\!/}\psi\;,$$

$$\gamma^i = e^{-\frac{1}{4}Z B\!\!\!/}\left(e^{-ZB}\right)^i{}_j\;\Gamma^j\;e^{\frac{1}{4}Z B\!\!\!/},$$

$$\varepsilon = e^{-\frac{1}{4}Z B\!\!\!/}\epsilon$$

$D = 3$ theory with 8 scalars and one fermion and $N = 8$ supersymmetry in adapted coordinates:

$$\mathcal{L}_{M2} = -\tfrac{1}{2}\Lambda\ \partial_\mu Z \partial^\mu Z \ - \ \tfrac{1}{2}\partial_\mu \chi^i \partial^\mu \chi^i \ - \ \bar{\theta} \partial\!\!\!/ \theta$$

$$- \ \partial_\mu \chi^i B_{ij} \chi^j \ \partial^\mu Z \ - \ \tfrac{1}{4}\bar{\theta}\ \partial\!\!\!/ Z\ B\!\!\!/\ \theta$$

$$\Lambda = 1 - \chi^i B_{ij} B_{jk} \chi^k$$

invariant under

$$\delta\chi^i = \bar{\varepsilon}\ \gamma^i\ \theta + \mathrm{i}\, B_{ij}\ \chi^j\ \bar{\varepsilon}\ \theta \qquad\qquad \delta Z = -\mathrm{i}\,\bar{\varepsilon}\ \theta,$$

$$\delta\theta = -\ \tfrac{1}{2}\gamma^i \partial\!\!\!/ \chi^i\ \varepsilon\ -\ \tfrac{1}{2}\,\mathrm{i}\,\partial\!\!\!/ Z\ \varepsilon$$
$$+\ \tfrac{1}{4}\,\mathrm{i}\,B\!\!\!/ \theta\ \bar{\varepsilon}\ \theta\ -\ \tfrac{1}{2}\gamma^i B_{ij} \chi^j \partial\!\!\!/ Z \varepsilon,$$

hence

$$\Rightarrow \begin{cases} [\delta_1, \delta_2] = -(\bar{\epsilon}_2 \tau^\mu \epsilon_1)\partial_\mu \\ \\ \delta \mathcal{L}_{M2} = 0 \end{cases}$$

taking into account that

$$\delta_1\ \varepsilon_2 \ = \ \delta_1\left(e^{-\frac{1}{4} Z B\!\!\!/}\epsilon_2\right) \ = \ \tfrac{1}{4}\,\mathrm{i}\,\bar{\varepsilon}_1 \theta\ B\!\!\!/ \varepsilon_2$$

Dualisation of Z into A_μ

Replace $\Sigma_\mu = \partial_\mu Z$ and impose Biancchi identity:

$$\mathcal{L}_{M2} = -\tfrac{1}{2}\Lambda\ \Sigma_\mu \Sigma^\mu\ -\ \tfrac{1}{2}\partial_\mu \chi^i \partial^\mu \chi^i\ -\ \bar{\theta}\partial\!\!\!/\theta$$

$$-\ \partial_\mu \chi^i B_{ij} \chi^j\ \Sigma^\mu\ -\ \tfrac{1}{4}\bar{\theta}\ \Sigma\!\!\!/\ B\!\!\!/\ \theta\ -\ \epsilon_{\mu\nu\rho} A_\mu \partial_\nu \Sigma_\rho$$

Dualisation condition:

$$\partial_\mu Z\ =\ -\tfrac{1}{2}\epsilon_{\mu\nu\kappa}\Lambda^{-1}\ F^{\nu\kappa}\ -\ \Lambda^{-1}\partial_\mu \chi^i B_{ij} \chi^j$$
$$-\ \tfrac{1}{4}\Lambda^{-1}\bar{\theta}\ \tau_\mu B\!\!\!/\ \theta$$

D2-brane action after dualisation yields

$$\mathcal{L}_{D2} = -\ \tfrac{1}{2}\partial_\mu \chi^i \partial^\mu \chi^i\ -\ \tfrac{1}{4}\Lambda^{-1}F^2 -\ \bar{\theta}\partial\!\!\!/\theta$$

$$-\ \tfrac{1}{4}\Lambda^{-1} B_{ij}\chi^i\ \bar{\theta}\ \partial\!\!\!/\chi^j B\!\!\!/\ \theta\ -\ \tfrac{1}{8}\Lambda^{-1}\bar{\theta} F\!\!\!/ B\!\!\!/\theta$$

$$-\ \tfrac{1}{2}\Lambda^{-1}\partial_\mu \chi^i B_{ij}\chi^j\ \chi^k B_{kl}\partial^\mu \chi^l$$

$$+\ \tfrac{1}{32}\Lambda^{-1}\bar{\theta}\ \tau_\mu B\!\!\!/\ \theta\ \bar{\theta}\ \tau^\mu B\!\!\!/\ \theta$$

$$-\ \tfrac{1}{2}\epsilon^{\mu\nu\kappa}\Lambda^{-1}\chi^i B_{ij}\partial_\mu \chi^j F_{\nu\kappa}$$

$\longrightarrow$ D2 probe in flux-brane background

$\longrightarrow$ How much supersymmetry is preserved?

Supersymmetry (breaking) of $\mathcal{L}_{D2}$

- Dualisation $\Longrightarrow$ $\mathcal{L}_{D2}$ independent of Z
- Replace $\partial_\mu Z$ by duality condition in supersymmetry rules of $\delta\chi^i$ and $\delta\theta$
- Truncate Z-dependence in $\varepsilon = e^{-\frac{1}{4} Z \not{B}}\epsilon$

$$\partial_Z\, \varepsilon = 0 \quad \Longrightarrow \quad \not{B}\, \epsilon = 0, \quad \varepsilon = \epsilon$$

$\longrightarrow$ *This breaks supersymmetry!*
$\longrightarrow$ *NB: Same condition as in backgrounds!*

- Apply duality condition to

$\delta\, \partial_\mu Z = -\,\mathrm{i}\,\bar{\epsilon}\, \partial_\mu \theta$

$$\Rightarrow \delta\, F_{\mu\nu} = -2\,\mathrm{i}\,\bar{\epsilon}\ \tau_{[\mu}\partial_{\nu]}\theta + 2\ \bar{\epsilon}\ \gamma^i \tau_{[\mu}\partial_{\nu]}\theta\ B_{ij}\chi^j + 2\ \bar{\epsilon}\ \gamma^i \tau_{[\mu}\ \theta\ B_{ij}\partial_{\nu]}\chi^j$$

$$\Rightarrow \delta A_\mu = \mathrm{i}\,\bar{\epsilon}\ \tau_\mu \theta\ -\ \bar{\epsilon}\ \tau_\mu \gamma^i\ \theta\ B_{ij}\chi^j$$

Supersymmetry rules for $D = 3$, $N = \not{B}\epsilon$ theory $\mathcal{L}_{D2}$ with 7 scalars, 1 vector and 1 fermion:

$$\delta\chi^i = \mathrm{i}\, B_{ij}\chi^j\ \bar{\epsilon}\theta\ +\ \bar{\epsilon}\,\gamma^i\,\theta$$

$$\delta A_\mu = \mathrm{i}\,\bar{\epsilon}\,\tau_\mu\theta\ -\ \bar{\epsilon}\,\tau_\mu\gamma^i\,\theta\ B_{ij}\chi^j\ ,$$

$$\begin{aligned}\delta\theta = &-\tfrac{1}{2}\gamma^i\not{\partial}\chi^i\epsilon\ +\ \tfrac{1}{2}\Lambda^{-1}\gamma^i\not{\partial}\chi^k\epsilon\ B_{ij}\chi^j\ B_{kl}\chi^l \\ &+\tfrac{1}{2}\,\mathrm{i}\,\Lambda^{-1}\not{\partial}\chi^i\epsilon\ B_{ij}\chi^j - \tfrac{1}{4}\Lambda^{-1}\gamma^i\not{F}\epsilon\ B_{ij}\chi^j \\ &-\tfrac{1}{4}\,\mathrm{i}\,\Lambda^{-1}\not{F}\epsilon\ +\ \tfrac{1}{4}\,\mathrm{i}\,\not{B}\theta\ \bar{\epsilon}\theta\ +\ \tfrac{1}{8}\,\mathrm{i}\,\Lambda^{-1}\tau^\mu\epsilon\ \bar{\theta}\tau_\mu\not{B}\theta \\ &+\tfrac{1}{8}\Lambda^{-1}\gamma^i\tau^\mu\epsilon\ \bar{\theta}\tau_\mu\not{B}\theta\ B_{ij}\chi^j\end{aligned}$$

Action $\mathcal{L}_{D2}$ invariant and supersymmetry algebra closes *up to* terms proportional to $\not{B}\epsilon$:

$$\delta\mathcal{L}_{D2} = 0 + (\ldots)\not{B}\epsilon$$

$$[\delta_1, \delta_2] = -(\bar{\epsilon}_2\tau^\mu\epsilon_1)\partial_\mu + (\ldots)\not{B}\epsilon$$

$B \in$	$\mathbb{1}$	$\mathfrak{sp}(1)$	$\mathfrak{su}(3)$	$\mathfrak{so}(7)$
Q	16	8	4	0
D2 in	M_{10}	$\mathcal{F}5$	$\mathcal{F}3$	"Melvin"

Compare to supersymm breaking in dimensional reduction and T-duality

Solution 1 $\longrightarrow$ (Dim. Reduction) $\longrightarrow$ Solution 2

$$\Longrightarrow \quad N(\text{Sol. } 1) \neq N\ (\text{Sol. } 2)$$

because $\mathcal{L}_\xi\ g_{\mu\nu} = 0$ but $\mathcal{L}_\xi \epsilon \neq 0$

E.g.: Reduction of Minkowski space over angular coordinate ϕ

[Bakas, 1994]

[Bergshoeff, Kallosh, Ortín, 1994]

N.B.: This is artefact of supergravity action!

String spectrum = massless modes + KK modes

Non-local world sheet effects $\Rightarrow$ non-local supersymmetry realisations

[Bakas, Sfetsos, 1995]

[Hassan, 1995]

[Alvarez, Alvarez-Gaumé, Bakas 1995]

[Sfetsos, 1995]

Action 1 $\longrightarrow$ (Hodgeduality) $\longrightarrow$ Action 2

$$\Longrightarrow \quad N(\text{Action 1}) \neq \; N\ (\text{Action 2})$$

because $\mathcal{L}_\xi\ g_{\mu\nu} = 0$ but $\mathcal{L}_\xi \epsilon \neq 0$

E.g.: Dualisation of $D = 3$, $N = 8$ theory with 8 scalars into $D = 3$, $N = \not{8}\epsilon$ with 7 scalars and 1 vector

N.B.: This is artefact of non-linear sigma model!

Born-Infeld action = non-lin. sigma model + higher order expansion

Non-local world sheet effects $\Rightarrow$ non-local supersymmetry realisations?

4. General considerations

$N = 8$ theory with 8 scalars:

$$\mathcal{L}_{M2} = -\frac{1}{2}\partial_\mu\varphi^a\partial^\mu\varphi^a + \bar{\psi}\partial\!\!\!/\psi$$

$$\delta\varphi^a = \bar{\epsilon}\,\Gamma^a\,\psi\,, \qquad \delta\psi = -\frac{1}{2}\Gamma^a\,\partial\!\!\!/\varphi^a\,\epsilon$$

Look for extra $U(1)$ symmetry

$$\delta_G\phi^a = \lambda\,\xi^a\,, \qquad \delta_G\psi = \lambda\,t\cdot\psi,$$

with $\quad \xi^a$: Killing vector, $\quad t$: realisation on fermions

$$t = t_0 + t_{(2)}\Gamma^{(2)} + t_{(4)}\Gamma^{(4)}$$

Question: for which ξ^a and t is (part of) the supersymmetry realised?

- $[\delta_G, \delta_Q] = \delta_Q(\eta)$

  $\Rightarrow \quad \delta_G\phi^a = C^a, \qquad \delta_G\psi = 0,$

  $\Rightarrow \xi^a$ is translation,

  $\Rightarrow$ standard dualisation / reduction case

- $\exists$ subgroup $\tilde{G} \subset G$ such that $[\delta_{\tilde{G}}, \delta_Q] = 0$?

$$[\delta_G, \delta_Q]\phi^a = -\lambda\bar{\psi}\Big[\Gamma^b \partial_b \xi^a - \\ -(t_{(0)} - t_{(2)} + t_{(4)})\Gamma^a\Big]\epsilon \equiv 0,$$

$$[\delta_G, \delta_Q]\psi = -\tfrac{1}{2}\lambda\Big[\Gamma^b \partial_b \xi^a + \\ +(t_{(0)} + t_{(2)} + t_{(4)})\Gamma^a\Big]\partial\!\!\!/\phi^a \epsilon \equiv 0,$$

$$\Longleftrightarrow \qquad t_{(0)} = t_{abcd} = 0$$

$$\Big[\Gamma^b \partial_b \xi^a + t_{bc}\Big(\Gamma^a\Gamma^{bc} - 4\delta^{ab}\Gamma^c\Big)\Big]\epsilon = 0$$

Solution: Killing vectors of the form

$$t_{ab} = -\tfrac{1}{4}\partial_{[a}\xi_{b]}, \qquad \Gamma^{ab} t_{ab}\,\epsilon = 0.$$

will commute with δ_Q and preserve $N = \not{t}\epsilon$ supersymmetry

Conclusions

- Standard reduction M2 $\rightarrow$ D2 world volume actions
- Dimensional reduction of Minkowski space gives (supersymmetric) fluxbranes for ξ = translation + rotation
- $D = 3$, $N = 8$ theory with 8 scalars (M2-probe in Minkowski) Hodge-dualised to $D = 3$, $N = \not{B}\epsilon$ theory with 7 scalars and 1 vector (D2-probe in fluxbrane background)
- General: supersymmetry of dualised theory = supersymmetry of original theory that commutes with Killing vector