A T-duality approach to the gravitational wave and the Kaluza-Klein monopole

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Abstract: We present a world volume action for the gravitational wave and the (bosonic) $N = 1$ Heterotic Kaluza-Klein monopole solution. In the construction of the action we make use of the fact that the wave and the Kaluza-Klein monopole are T-dual to the fundamental string and the solitonic five-brane, and we present an alternative way to derive the T-duality rules for non-string-like objects.

1. Introduction

Amongst the various extended object solutions of supergravity theories, there are two solutions that do not have the usual brane interpretation: the gravitational wave ($\mathcal{W}$) and the Kaluza-Klein monopole ($\mathcal{KK}$). Although they appear as solutions of supergravity theories, they are already present as solutions of pure gravity and exist in any dimension $D > 4$, the wave even for $D > 2$. In ten dimensions, the wave and the monopole are T-dual to the fundamental string ($F1$) and the solitonic five-brane ($S5$) respectively and their eleven dimensional counterparts are, via dimensional reduction, related to the $D0$-brane and the $D6$-brane [1].

It is natural to look for the effective action for these solutions: these effective actions describe the zero-modes of the solutions and occur as source terms in the supergravity equations of motion. The zero-modes of the eleven dimensional Kaluza-Klein monopole have been discussed in [2], and the kinetic term for the effective action was given in [3]. In this paper, we will show that the effective action of the gravitational wave is given by the one of a massless particle and construct explicitly the effective action for the ten-dimensional $N = 1$ Heterotic Kaluza-Klein monopole. In both cases we will make use of the T-duality relations with the fundamental string and the solitonic five-brane, though a different way of deriving the T-duality rules has to be introduced: T-duality via dimensional reduction. This paper is organised as follows: in Section 2 we will present the idea of T-duality via dimensional reduction and derive the duality rules. In Section 3 these rules are used to relate the effective action of the gravitational wave to the action of the fundamental string and in Section 4 we explicitly construct an effective action for the $N = 1$ Heterotic monopole solution.

2. T-duality via dimensional reduction

Target space duality, or for short T-duality, is a symmetry transformation that relates different string backgrounds to each other and was first introduced at the level of the bosonic sigma model for a fundamental string, in the presence of an isometry direction [4]. Starting from a sigma model, consisting of the embedding coordinates $X^{\hat{\mu}}$, a metric $\hat{g}_{\hat{\mu}\hat{\nu}}$, an axion $\hat{B}_{\hat{\mu}\hat{\nu}}$ and a dilaton $\hat{\phi}$, and with an isometry direction lying in the x-direction (so all fields are independent of x), one can split the $X^{\hat{\mu}}$ into $X^{\hat{\mu}} = (X^\mu, X)$ and the target space indices $\hat{\mu}$ into $\hat{\mu} = (\mu, x)$. Dualization of the embedding coordinate X on the world sheet yields a new world sheet scalar $\tilde{X}$ and the newly obtained action, in terms of the dual coordinates $\tilde{X}^{\hat{\mu}} = (X^\mu, \tilde{X})$, is again a bosonic sigma model, with a dual metric $\tilde{\hat{g}}_{\hat{\mu}\hat{\nu}}$, axion $\tilde{\hat{B}}_{\hat{\mu}\hat{\nu}}$ and dilaton $\tilde{\hat{\phi}}$. The relations between the original and the dual background fields are called the T-duality rules and are given by [4]:

$$
\begin{array}{ll}
\tilde{\hat{g}}_{xx} = 1/\hat{g}_{xx}, & \tilde{\hat{B}}_{x\mu} = \hat{g}_{x\mu}/\hat{g}_{xx}, \\
\tilde{\hat{g}}_{x\mu} = \hat{B}_{x\mu}/\hat{g}_{xx}, & \tilde{\hat{B}}_{\mu\nu} = \hat{B}_{\mu\nu} - (\hat{g}_{x\mu}\hat{B}_{x\nu} - \hat{g}_{x\nu}\hat{B}_{x\mu})/\hat{g}_{xx}, \\
\tilde{\hat{g}}_{\mu\nu} = \hat{g}_{\mu\nu} - (\hat{g}_{x\mu}\hat{g}_{x\nu} - \hat{B}_{x\mu}\hat{B}_{x\nu})/\hat{g}_{xx}, & \tilde{\hat{\phi}} = \hat{\phi} - \tfrac{1}{2}\log|\hat{g}_{xx}|.
\end{array}
\tag{1}
$$

Note, however, that in the derivation of these duality rules we made intrinsically use of the fact that we are dealing with a string-like solution: only on a two-dimensional world volume a scalar X can be dualized to another scalar $\tilde{X}$. It is not clear how to generalize this procedure to more general extended objects. In particular, if we later on want to use the T-duality between the actions of the solitonic five-brane and the Kaluza-Klein monopole, we first have to find a way to properly construct the duality rules between these solitons.

Another way to derive the T-duality between extended objects is to make use of the fact that all fields have an isometry direction x and to perform a dimensional reduction over this x-direction. Then two solutions can said to be T-dual to each other, if they can be mapped onto the same solution, one dimension lower, via different dimensional reduction schemes. The T-duality rules, obtained by relating the two reduction schemes, map one solution onto the other. This is similar to the way Type II T-duality rules between the Type IIA and Type IIB supergravity actions was derived for the R-R fields [5].

As an example, let us look at the T-duality between the fundamental string and the gravitational wave. The relation between the ten and nine-dimensional fields are given by

$$
\begin{array}{rclrcl}
\hat{g}_{\mu\nu} & = & g_{\mu\nu} - k^2 A_\mu A_\nu\,, & \hat{B}_{\mu\nu} & = & B_{\mu\nu} + A_{[\mu}B_{\nu]}\,, \\
\hat{g}_{x\mu} & = & -k^2 A_\mu\,, & \hat{B}_{x\mu} & = & B_\mu\,, \\
\hat{g}_{xx} & = & -k^2\,, & \hat{\phi} & = & \phi + \tfrac{1}{2}\log k.
\end{array}
\tag{2}
$$

such that the double dimensional reduction of the fundamental string solution

$$
F1 = \begin{cases} ds^2 = H^{-1}(dt^2 - dx_1^2) - (dx_2^2 + ... + dx_9^2) \\ e^{-2\hat{\phi}} = H \\ \hat{B}_{01} = H^{-1} \end{cases}
\tag{3}
$$

over the world volume direction x_1 gives rise to a nine-dimensional point particle solution [6]

$$
m0 = \begin{cases} ds^2 = H^{-1}dt^2 - (dx_2^2 + ... + dx_9^2), \\ k = H^{-1/2}, \qquad e^{-2\phi} = H^{1/2}, \\ B_0 = -H^{-1}, \qquad B_{\mu\nu} = A_\mu = 0\ . \end{cases}
\tag{4}
$$

The direct dimensional reduction of the gravitational wave

$$
\mathcal{W}\ :\ ds^2 = (2-H)dt^2 - Hdz^2 + 2(1-H)dtdz - (dx_2^2 + ... + dx_9^2)
\tag{5}
$$

over the propagation direction z yields

$$
\tilde{m0} = \begin{cases} d\tilde{s}^2 = H^{-1}dt^2 - (dx_2^2 + ... + dx_9^2), \\ \tilde{k} = H^{1/2}, \qquad e^{-2\tilde{\phi}} = H^{1/2}, \\ \tilde{A}_0 = -H^{-1}, \qquad \tilde{B}_{\mu\nu} = \tilde{B}_\mu = 0\ . \end{cases}
\tag{6}
$$

We see that these two point particle solutions (4) and (6) are actually the same, if we identify

$$
\tilde{A}_\mu = B_\mu, \qquad \tilde{B}_\mu = A_\mu, \qquad \tilde{k} = 1/k.
\tag{7}
$$

Rewritten in terms of the ten-dimensional fields via (2), this corresponds precisely to the T-duality rules (1).

3. Wave/string duality

The T-duality between the wave/string solutions suggests that a same type of duality exists between the world volume actions that describe the dynamics of these solutions. We will use the procedure of T-duality via dimensional reduction to construct the duality map between the actions.

Our starting point is the world volume action for the fundamental string, consisting of a kinetic term in the Nambu-Goto form, together with a Wess-Zumino term:

$$\mathcal{L}_{F1} \;=\; \sqrt{|\det(\partial_i \hat{X}^{\hat{\mu}} \partial_j \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}})|} \;+\; \tfrac{1}{2}\, \varepsilon^{ij} \partial_i \hat{X}^{\hat{\mu}} \partial_j \hat{X}^{\hat{\nu}} \hat{B}_{\hat{\mu}\hat{\nu}} \tag{8}$$

The indices $i,\ j$ are the world volume indices $\tau,\ \sigma$. If we assume that the direction in which the string is oriented is compact and the string is wound m times around this direction, we can make a split in the target space coordinates as follows:

$$\hat{X}^{\hat{\mu}}(\tau,\sigma) = \big(X^{\mu}(\tau),\ m\sigma\big). \tag{9}$$

This means that we have identified the world volume direction of the string with the space-time direction in which the string is oriented. Using this split in the coordinates and the reduction rules given in (2), we can rewrite the action (8) after double dimensional reduction as

$$\mathcal{L}_{m0} \;=\; mk\, \sqrt{|\det(\partial X^{\mu} \partial X^{\nu} g_{\mu\nu})|} \;-\; m\, \partial X^{\mu} B_{\mu}, \tag{10}$$

which is the action for a massive particle with mass m. With ∂X^{μ} we mean the partial derivative of X^{μ} with respect to τ.

The world volume action of a gravitational wave is given by

$$\mathcal{L}_{\mathcal{W}} = \tfrac{1}{2}\sqrt{|\gamma|}\, \gamma^{-1}\, \partial \hat{X}^{\hat{\mu}}\, \partial \hat{X}^{\hat{\nu}}\, \hat{g}_{\hat{\mu}\hat{\nu}}. \tag{11}$$

Direct dimensional reduction gives an action of the form

$$\mathcal{L} = \tfrac{1}{2}\sqrt{|\gamma|}\, \gamma^{-1} \Big[\partial X^{\mu} \partial X^{\nu} g_{\mu\nu} \;-\; \tilde{k}^2 (\partial S + \tilde{A}_{\mu} \partial X^{\mu})^2\Big], \tag{12}$$

where S is the world sheet scalar coming from the compact dimension: $S = \hat{X}^x$. It can be eliminated via its equations of motion

$$\partial S = \sqrt{|\gamma|}\, \mu\, \tilde{k}^{-2} - \tilde{A}_{\mu} \partial X^{\mu}, \tag{13}$$

where μ is a constant that corresponds to the momentum in the compactified direction. Before substituting this expression directly in (11), we have to check whether this equation is consistent with the other equations of motion. The substitution can be done if we add to the Lagrangian (12) a total derivative term $\mu\partial S$ [7, 8], giving

$$\mathcal{L} = \tfrac{1}{2}\sqrt{|\gamma|}\gamma^{-1} \partial X^{\mu} \partial X^{\nu} g_{\mu\nu} \;+\; \tfrac{1}{2}\sqrt{|\gamma|}\, \mu^2 \tilde{k}^{-2} \;-\; \mu\, \tilde{A}_{\mu} \partial X^{\mu}. \tag{14}$$

Eliminating the world line metric γ via its equation of motion, we can go to the Nambu-Goto form and find the action for a massive particle with mass μ:

$$\mathcal{L}_{m\tilde{0}} = \mu\, \tilde{k}^{-1} \sqrt{|\det(\partial X^{\mu} \partial X^{\nu} g_{\mu\nu})|} \;-\; \mu\, \partial X^{\mu} \tilde{A}_{\mu}, \tag{15}$$

which is, up to (7), exactly the same Lagrangian that was obtained via T-dual dimensional reduction from the fundamental string, provided that we identify the constants m and μ. Physically this means that the T-duality interchanges the winding number of the string with the momentum of the wave in the dualized direction.

4. Five-brane/monopole duality

The solitonic five-brane ($S5$) and the Kaluza-Klein monopole ($\mathcal{KK}$) solutions are given by

$$S5 \quad = \quad \begin{cases} ds^2 = dt^2 - dx_1^2 - \ldots - dx_5^2 - H(dx_6^2 + \ldots + dx_9^2) \\ e^{-2\phi} = H^{-1} \\ H_{mnp} = \varepsilon_{mnpr}\partial_r H \qquad\qquad\qquad (m,n,p,r:6,\ldots,9), \end{cases} \tag{16}$$

$$\mathcal{KK} \quad = \quad \begin{cases} ds^2 = dt^2 - dx_1^2 - \ldots - dx_5^2 - H^{-1}(dz + A_m dx_m)^2 - H dx_m^2 \\ F_{mn} = \varepsilon_{mnp}\,\partial_p H \qquad\qquad\qquad (m,n,p:7,\ldots,9) \end{cases} . \tag{17}$$

The harmonic function H depends on the transverse directions x^m and is, in the case of the monopole, independent of z. The T-duality between these two solutions can be seen by doing a dimensional reduction of the $S5$ over an transverse direction x^m and of the $\mathcal{KK}$ over the z-direction, or by directly applying the duality rules (1).

Let us now make use of the T-duality between the $S5$ and the $\mathcal{KK}$ in order to construct a world volume action for the monopole. We will present here the bosonic part of the world volume action for the Heterotic ($N = 1$ supersymmetric) monopole. For the kinetic term in the action of the $D = 11$ monopole and the $D = 10$ Type IIA monopole we refer to [3].

Our strategy will be similar to the one in the previous section: because of the T-duality we know that a correct action for the monopole is one that, upon a T-dual compactification, reduces to the nine-dimensional form of the $S5$. The reduction of the $N = 1$ five-brane world volume action is straightforward. Starting from the action

$$S_{(S5)} = -\tfrac{T}{2}\int d^6\sigma\; e^{-2\hat{\phi}}\sqrt{|\det(\partial_i \hat{X}^{\hat{\mu}}\partial_j \hat{X}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}})|} \quad + \quad \tfrac{T}{6!}\int d^6\sigma\; \varepsilon^{i_1\ldots i_6}\,\partial_{i_1}\hat{X}^{\hat{\mu}_1}\ldots\,\partial_{i_6}\hat{X}^{\hat{\mu}_6}\hat{C}_{\hat{\mu}_1\ldots\hat{\mu}_6}\ , \tag{18}$$

and using the reduction rules

$$\begin{array}{ll} \hat{X}^{\mu} = X^{\mu}\,, & \hat{X}^{x} = S \\ \hat{g}_{\mu\nu} = g_{\mu\nu} - k^2 A_\mu A_\nu\,, & \hat{C}_{x\mu_1\ldots\mu_5} = D_{\mu_1\ldots\mu_5},, \\ \hat{g}_{x\mu} = -k^2 A_\mu\,, & \hat{C}_{\mu_1\ldots\mu_6} = C_{\mu_1\ldots\mu_6} + 6A_{[\mu_1}D_{\mu_2\ldots\mu_6]}\,, \\ \hat{g}_{xx} = -k^2\,, & \hat{\phi} = \phi + \tfrac{1}{2}\log k, \end{array} \tag{19}$$

we find for the reduced five-brane action

$$S_{rS5} = -\tfrac{T}{2}\int d^6\sigma\; e^{-2\phi}k^{-1}\sqrt{|\det(g_{ij} - k^2 F_i F_j)|} \quad + \quad \tfrac{T}{6!}\int d^6\sigma\; \varepsilon^{i_1\ldots i_6}\,\Big[C_{i_1\ldots i_6} - 6\; D_{i_1..i_5}F_{i_6}\Big], \tag{20}$$

where

$$g_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}, \qquad\qquad F_i = \partial_i S + A_\mu \partial_i X^\mu. \tag{21}$$

Our task is now to find an action for the monopole that, upon the reduction (19) gives an action that is T-dual to (20), or equivalently, gives the same action upon T-dual reduction.

However, due to the presence of the isometry direction z in the monopole solution, a subtlety occurs in the counting of the degrees of freedom: on the one hand, it turns out that this z-direction can not be interpreted as a world volume direction [2]. So, of the ten embedding coordinates $\hat{X}^{\hat{\mu}}$ only the six $\hat{X}^i$ corresponding to the six world volume directions can be eliminated as degrees of freedom, by fixing the diffeomorphisms on the $\hat{X}^i$. Naively we would be with left four scalar degrees of freedom: the three transversal coordinates $\hat{X}^m$ and the isometry direction Z. However, on the other hand, being an isometry direction, Z does not count as a degrees of freedom (translations along the z-direction give the same metric again).

So in one way or another we have to get rid of this extra degree of freedom Z, at the same time introducing another scalar in order to construct an $N=1, D=6$ scalar multiplet. We will conceive this via a *gauged sigma model*, where we will gauge the isometry direction, eliminating the Z degree of freedom, and introducing a scalar $\hat{S}$ to get the counting right [3].

Our proposal for the kinetic term of the monopole action is:

$$S_{\mathcal{KK}} = \tfrac{T}{2} \int d^6\sigma \ \hat{k}^2 \ e^{-2\hat{\phi}} \ \sqrt{\left|\det\left(\partial_i \hat{X}^{\hat{\mu}} \partial_j \hat{X}^{\hat{\nu}} \hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}} - \hat{k}^{-2} \hat{\mathcal{F}}_i \hat{\mathcal{F}}_j\right)\right|}. \tag{22}$$

The vector $\hat{k}^{\hat{\mu}}$ a Killing vector associated with the isometry direction z, such that

$$\hat{k}^2 = -\hat{k}^{\hat{\mu}} \hat{k}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}. \tag{23}$$

In coordinates adapted to the isometry, $\hat{k}^{\hat{\mu}}$ will be of the form $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu} z}$. Furthermore we introduced a "metric" $\hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}}$ and a scalar $\hat{S}$ via

$$\hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}} = \hat{g}_{\hat{\mu}\hat{\nu}} \ + \ \hat{k}^{-2} \ \hat{k}_{\hat{\mu}} \ \hat{k}_{\nu}, \qquad \hat{\mathcal{F}}_i = \partial_i \hat{S} \ + \ \partial_i \hat{X}^{\hat{\mu}} \ \hat{k}^{\hat{\nu}} \ \hat{B}_{\hat{\mu}\hat{\nu}}. \tag{24}$$

The action (22) is a gauged sigma model, because of the symmetries

$$\delta \hat{X}^{\hat{\mu}} = \Lambda \ \hat{k}^{\hat{\mu}}, \qquad \delta \hat{S} = -\hat{k}^{\hat{\mu}} \Sigma_{\hat{\mu}}, \qquad \delta \hat{B}_{\hat{\mu}\hat{\nu}} = \partial_{[\hat{\mu}} \hat{\Sigma}_{\hat{\nu}]}, \tag{25}$$

which corresponds to a gauging of the isometry direction Z. This symmetry occurs because of the presence of the Killing vector $\hat{k}^{\hat{\mu}}$, which effectively projects the z-direction and the corresponding field $Z(\sigma)$ out of the action. This can be seen in the contraction of the $\hat{k}^{\hat{\mu}}$ with the "metric" $\hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}}$:

$$\begin{aligned} \hat{k}^{\hat{\mu}} \ \hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}} &= \hat{k}^{\hat{\mu}} \hat{g}_{\hat{\mu}\hat{\nu}} + \hat{k}^{-2} \hat{k}^{\hat{\mu}} \hat{k}_{\hat{\mu}} \hat{k}_{\hat{\nu}} \\ &= \hat{k}^{\hat{\mu}} \hat{g}_{\hat{\mu}\hat{\nu}} - \hat{k}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\nu}} \ = 0. \end{aligned} \tag{26}$$

The "metric" $\hat{\mathcal{G}}_{\hat{\mu}\hat{\nu}}$ is therefore effectively a nine-dimensional metric, written in a ten-dimensional covariant form. Reducing the action (22) over the isometry direction, using the reduction rules (2), we find for the reduced monopole action

$$S = -\tfrac{T}{2} \int d^6\sigma \ e^{-2\phi} \tilde{k} \ \sqrt{\left|\det\left(g_{ij} - \tilde{k}^{-2} (\partial_i S + \tilde{B}_\mu \partial_i X^\mu)(\partial_j S + \tilde{B}_\nu \partial_j X^\nu.)\right)\right|} \ , \tag{27}$$

This is indeed precisely the action (20) of the reduced five-brane up to a T-duality transformation (7).

Our Ansatz for the ten-dimensional Wess-Zumino term of the monopole is:

$$S = \tfrac{T}{6!} \int d^6\sigma \varepsilon^{i_1 \cdots i_6} \hat{k}^{\hat{\mu}_1} \ \partial_{i_1} \hat{X}^{\hat{\mu}_2} ... \partial_{i_6} \hat{X}^{\hat{\mu}_7} \left[\hat{A}_{\hat{\mu}_1 \cdots \hat{\mu}_7} - 6 \hat{D}_{\hat{\mu}_1 \cdots \hat{\mu}_6} (\partial_{\hat{\mu}_7} \hat{S} - \hat{k}^{\hat{\nu}} \hat{B}_{\hat{\mu}_7 \hat{\nu}})\right], \tag{28}$$

where

$$\begin{aligned} \partial_{[\hat{\mu}_1} \hat{A}_{\hat{\mu}_2 \ldots \hat{\mu}_8]} &= \tfrac{1}{2!7!\sqrt{|\hat{g}|}} \hat{k}^2 e^{-2\hat{\phi}} \varepsilon_{\hat{\mu}_1 \ldots \hat{\mu}_{10}} [\hat{k}^2 \hat{F}^{\hat{\mu}_9 \hat{\mu}_{10}}(\hat{A}) + 2 \hat{k}^{\hat{\nu}} \hat{B}_{\hat{\nu}\hat{\rho}} \hat{H}^{\hat{\rho}\hat{\mu}_9 \hat{\mu}_{10}}], \\ \partial_{[\hat{\mu}_1} \hat{D}_{\hat{\mu}_2 \ldots \hat{\mu}_7]} &= \tfrac{1}{8!\sqrt{|\hat{g}|}} e^{-2\hat{\phi}} \varepsilon_{\hat{\mu}_1 \ldots \hat{\mu}_{10}} \hat{H}^{\hat{\mu}_8 \hat{\mu}_9 \hat{\mu}_{10}}, \\ \hat{A}_{\hat{\mu}} &= \hat{k}^{-2} \hat{k}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}} \ . \end{aligned} \tag{29}$$

The vector $\hat{A}_{\hat{\mu}}$ is the uplifting of the Kaluza-Klein vector A_μ, which can be written in a ten-dimensional form via the Killing vector $\hat{k}^{\hat{\mu}}$. The seven-form $\hat{A}_{\hat{\mu}_1 \ldots \hat{\mu}_7}$ and the six-form $\hat{D}_{\hat{\mu}_1 \ldots \hat{\mu}_6}$ are the Poincaré duals of $\hat{A}_{\hat{\mu}}$ and $\hat{B}_{\hat{\mu}\hat{\nu}}$ respectively.

The Wess-Zumino term (28) transforms as a total derivative under the gauge transformations

$$\begin{aligned} &\delta \hat{A}_{\hat{\mu}_1 \ldots \hat{\mu}_7} = \partial_{[\hat{\mu}_1} \Lambda_{\hat{\mu}_2 \ldots \hat{\mu}_7]} + \partial_{[\hat{\mu}_1} \Gamma_{\hat{\mu}_2 \ldots \hat{\mu}_6} \hat{k}^{\hat{\nu}} \hat{B}_{|\hat{\nu}|\hat{\mu}_7]} \, , \qquad && \delta \hat{S} = -\hat{k}^{\hat{\mu}} \Sigma_{\hat{\mu}} \\ &\delta \hat{D}_{\hat{\mu}_1 \ldots \hat{\mu}_6} = \partial_{[\hat{\mu}_1} \Gamma_{\hat{\mu}_2 \ldots \hat{\mu}_6]}, && \delta \hat{B}_{\hat{\mu}\hat{\nu}} = \partial_{[\hat{\mu}} \Sigma_{\hat{\nu}]}. \end{aligned} \tag{30}$$

Again the contractions with the Killing vector $\hat{k}^{\hat{\mu}}$ take care of the fact that the z-direction is projected out of the action (28). Reduction over the isometry direction z gives

$$S = \tfrac{T}{6!} \int d^6\sigma\ \varepsilon^{i_1 \dots i_6}\ \partial_{i_1} X^{\mu_1} \dots \partial_{i_6} X^{\mu_6} \Big[\tilde{A}_{\mu_1 \dots \mu_6} - 6\ D_{\mu_1 \dots \mu_5}(\partial_{\mu_6} S + \tilde{B}_{\mu_6})\Big], \tag{31}$$

which can be mapped onto the Wess-Zumino term of the $S5$-brane, via the T-duality transformation

$$\tilde{B}_\mu = A_\mu, \qquad \tilde{A}_{\mu_1 \dots \mu_6} = C_{\mu_1 \dots \mu_6}. \tag{32}$$

We see that in $D = 9$ not only the Kaluza-Klein vector A_μ and the winding vector B_μ get interchanged by T-duality, but also their Poincaré duals $A_{\mu_1 \dots \mu_6}$ and $C_{\mu_1 \dots \mu_6}$.

The kinetic term (22) and the WZ-term (28) together form the full world volume action for the $N = 1$ Heterotic Kaluza-Klein monopole in ten dimensions. It would be interesting to construct the full action for the $N = 2$ monopole of Type IIA/B theory and of the Kaluza-Klein monopole of eleven-dimensional supergravity, which is related to the $D6$-brane via dimensional reduction.

Gauged sigma models have been used recently in [9] to describe the eleven-dimensional origin of brane-solutions in massive Type IIA theory. Given this, it is logical to expect that the action of the monopole in a massive background is given by a sigma model which is gauged in two different directions: one gauging of the isometry direction z, as performed above, and an extra gauging in order to obtain the monopole in a massive background. It would be interesting to see whether the double gauged sigma model can indeed be fitted into massive eleven dimensional supergravity, following the procedures of [9].

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