Einstein branes

Bert Janssen

Universidad de Granada & CAFPE

In collaboration with: W. Chemissany (K.U. Leuven) and T. Van Riet (U. Uppsala)

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Outlook

1. Introduction: flat p-branes and their curved generalizations

$$d\hat{s}^2 = H^A(r)\,\tilde{g}_{ij}\,dx^i dx^j \; + \; H^B(r)\left[dr^2 + r^2 d\Omega_n^2\right] \qquad \tilde{R}_{ij} = p\,\tilde{\Lambda}\,\tilde{g}_{ij}$$

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- fundamental objects in string theory
- solitonic solutions in supergravity

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The importance of p-branes in string theory can hardly be overestimated:

- fundamental objects in string theory
- solitonic solutions in supergravity
- crucial for understanding of
  - gauge/gravity duals
  - black hole entropy
  - string phenomenology & cosmology
  - non-perturbative effects
  - ...

In supergravity: solutions of

$$S_D = \int \mathrm{d}^D x \sqrt{|\hat{g}|} \left[\hat{R} - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2(p+2)!} \, \mathrm{e}^{b\hat{\phi}} \hat{F}_{\hat{\mu}_1 \ldots \hat{\mu}_{p+2}} \hat{F}^{\hat{\mu}_1 \ldots \hat{\mu}_{p+2}} \right]$$

of the form

$$\mathrm{d}\hat{s}^2 = H^{2A}(r)\, \eta_{ij}\, \mathrm{d}x^i \mathrm{d}x^j + H^{2B}(r) \Big(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_n^2 \Big),$$

$$e^{-2\hat{\phi}} = H^C(r), \qquad \hat{F}_{i_1 \ldots i_{p+1} r} = \partial_r H^E(r)\, \varepsilon_{i_1 \ldots i_{p+1}}$$

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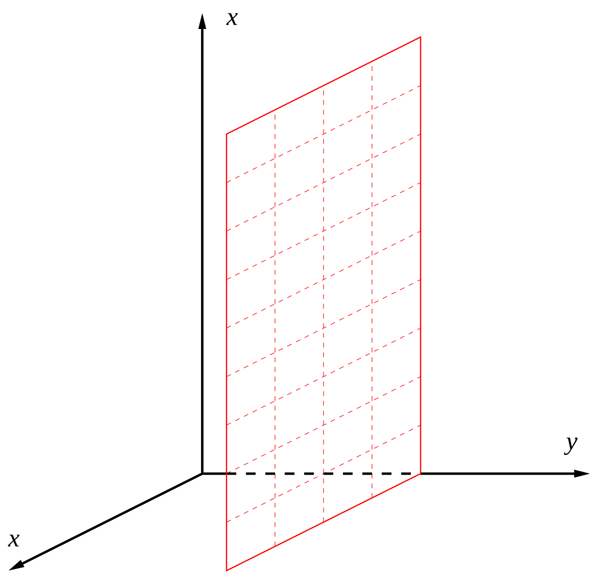

- Planar extended objects with p spatial directions
- Electrically and/or magnetically charged under $\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}$
- $D = 11,\ p = 2$: M2 and M5-brane
  $D = 10,\ p = 1$: F1 and NS5-brane
  $D = 10,\ 0 \leq p \leq 8$: Dp-branes

[Duff et al.] [Güven] [Dahbolkar et al.] [Callan, et al.] [Duff et al.] [Polchinski]

General solution:

$$\mathrm{d}\hat{s}^2 = H^{2A}(r)\,\eta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + H^{2B}(r)\Big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_n^2\Big),$$

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with

$$A = -\frac{4(D-p-3)}{\Delta(D-2)} \qquad B = \frac{4(p+1)}{\Delta(D-2)} \qquad C = \frac{4b}{\Delta}$$

$$\Delta = b^2 + 2\frac{(p+1)(D-p-3)}{D-2} \qquad \nabla^2 H(r) = 0$$

[Stelle][Bergshoeff]

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Known generalisations:

- Ricci-flat branes [Brecher et al.] [B.J.] [Figueroa-O'Farrill]
- Curved domain walls [Kachru et al.] [Alonso-Alberca et al.] ...

Ricci-flat branes:

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Reason: factorisation of x-dependence

$$R_{ij} = \tilde{R}_{ij} - \tilde{g}_{ij} H^{2(A-B)-1}\Big[\nabla^2 H + (\partial H)^2\Big]$$

$\longrightarrow$ eqn of motions:

$$\tilde{R}_{ij} - \tilde{g}_{ij} H^{2(A-B)-1}\Big[\nabla^2 H + (\partial H)^2\Big] + \tilde{g}_{ij}(\partial\hat{\phi})^2 + \tilde{g}_{ij}\,e^{b\hat{\phi}}\,\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\hat{F}^{\hat{\mu}_1\ldots\hat{\mu}_{p+2}} = 0$$

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No known solutions of general p-branes with $\tilde{R}_{ij} = p\tilde{\Lambda}\, \tilde{g}_{ij}$ (Einstein branes)

$\longrightarrow$ Knippenberg (2003): such solutions do not exist

End of the story (?)

One to one correspondence between p-brane and DW

$$d\hat{s}^2 \;=\; e^{2A(r)}\,\tilde{g}_{ij}(x)\,dx^i dx^j \;+\; e^{2B(r)}\Big(dr^2 + r^2 d\Sigma_n^2\Big)$$

$$ds^2 \;=\; a^2(z)\tilde{g}_{ij}(x)\,dx^i dx^j \;+\; f^2(z)dz^2$$

through dimensional reduction over angular part $d\Sigma_n^2 = \bar{h}_{ab}(\theta)d\theta^a d\theta^b$ with

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$\longrightarrow$ Knippenberg's argument invalidated: too restrictive Ansatz?

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- We have a non-compactification:

$$d\hat{s}^2 = e^{2A(r)}\, \tilde{g}_{ij}\, dx^i dx^j + e^{2B(r)} \Big(dr^2 + r^2 d\Sigma_n^2 \Big)$$

2. Reduction to domain wall problem

Magnetic Ansatz:

$$\mathrm{d}\hat{s}^2 \;=\; \mathrm{e}^{2\alpha\chi}\, g_{\mu\nu}\, \mathrm{d}x^\mu \mathrm{d}x^\nu \;+\; \mathrm{e}^{2\beta\chi}\, \bar{h}_{ab}\, \mathrm{d}\theta^a \mathrm{d}\theta^b$$

$$\hat{\phi} = \phi(x), \qquad\qquad \hat{F}_{a_1 \ldots a_n} \;=\; \frac{1}{\sqrt{|\bar{h}|}}\, Q\, \bar{\varepsilon}_{a_1 \ldots a_n}$$

with $\mu, \nu \in \{0, \ldots, p+2\}$, $a, b \in \{1, \ldots, n\}$

$$g_{\mu\nu} = g_{\mu\nu}(x), \qquad \chi = \chi(x), \qquad \bar{h}_{ab} = \bar{h}_{ab}(\theta), \qquad \bar{R}_{ab} = (n-1)\bar{K}\bar{h}_{ab}$$

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Nota bene:

- $\bar{h}_{ab}$ is metric of angular part of transverse space
- χ is breathing mode
- $g_{\mu\nu}$ is metric of domain wall spacetime

$$ds^2 \;=\; g_{\mu\nu}\, dx^\mu dx^\nu \;=\; a^2(z)\tilde{g}_{ij}(x)\, dx^i dx^j \;+\; f^2(z) dz^2$$

More precisely:

$$
\begin{aligned}
\mathrm{d}\hat{s}^2 &= \mathrm{e}^{2A(r)}\,\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j \;+\; \mathrm{e}^{2B(r)}\Big(\mathrm{d}r^2 + r^2\bar{h}_{ab}\,\mathrm{d}\theta^a\mathrm{d}\theta^b\Big) \\
&= \mathrm{e}^{2A(r)}\,\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j \;+\; \mathrm{e}^{2B(r)}\mathrm{d}r^2 \;+\; \mathrm{e}^{2B(r)}r^2\bar{h}_{ab}\,\mathrm{d}\theta^a\mathrm{d}\theta^b \\
&= \mathrm{e}^{2\alpha\chi}\Big[a^2(z)\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j \;+\; f^2(z)\mathrm{d}z^2\Big] \;+\; \mathrm{e}^{2\beta\chi}\,\bar{h}_{ab}\,\mathrm{d}\theta^a\mathrm{d}\theta^b \\
&= \mathrm{e}^{2\alpha\chi}\,g_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu \;+\; \mathrm{e}^{2\beta\chi}\,\bar{h}_{ab}\,\mathrm{d}\theta^a\mathrm{d}\theta^b
\end{aligned}
$$

provided that

$$
\mathrm{e}^{\beta\chi(z)} = \mathrm{e}^{B(r)}r, \qquad \mathrm{e}^{\alpha\chi(z)}\,a(z) = \mathrm{e}^{A(r)}, \qquad \mathrm{e}^{\alpha\chi(z)}\,f(z)\,\mathrm{d}z = \mathrm{e}^{B(r)}\,\mathrm{d}r
$$

Dimensional reduction: $(\tilde{D} = p\alpha + n\beta)$

$$\begin{aligned} \hat{R} &= \mathrm{e}^{-2\alpha\chi} R \, + \, n(n-1)\mathrm{e}^{-2\beta\chi}\bar{K} \\ & \quad + 2(\tilde{D}+\alpha)\mathrm{e}^{-2\alpha\chi}\nabla^2\chi \, + \, \Big[\tilde{D}^2 + p\alpha^2 + n\beta^2\Big]\mathrm{e}^{-2\alpha\chi}(\partial\chi)^2 \end{aligned}$$

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hence

$$S \;=\; \int \mathrm{d}^D x\sqrt{|\hat{g}|}\,\Big[\hat{R} \;-\; \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2(p+2)!}\,\mathrm{e}^{b\hat{\phi}}\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\hat{F}^{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\Big]$$

$$= \int \mathrm{d}^{p+2}x\sqrt{|g|}\Big[R \;-\; \tfrac{1}{2}(\partial\phi)^2 \;-\; \tfrac{1}{2}(\partial\chi)^2 \;-\; \tfrac{1}{2}\mathrm{e}^{b\phi+c\chi}Q^2 \;+\; n(n-1)e^{d\chi}\bar{K}\Big]$$

provided that $\quad p\alpha = -n\beta \quad$ and $\quad \alpha = \sqrt{\frac{n}{2p(p+n)}}$

$$\Longrightarrow \quad \beta = \beta(p,n), \qquad c = c(p,n), \qquad d = d(p,n)$$

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$$=\; \int \mathrm{d}^{p+2}x\sqrt{|g|}\Big[R \;-\; \tfrac{1}{2}(\partial\phi)^2 \;-\; \tfrac{1}{2}(\partial\chi)^2 \;-\; \tfrac{1}{2}\mathrm{e}^{b\phi+c\chi}Q^2 \;+\; n(n-1)e^{d\chi}\bar{K}\Big]$$

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$\longrightarrow$ gravity coupled to 2 scalars in double exponential potential

Curved domain wall Ansatz:

$$ds^2 = a^2(z)\tilde{g}_{ij}(x)\,dx^i dx^j + f^2(z)dz^2$$

$$\phi = \phi(z), \qquad \chi = \chi(z), \qquad \tilde{R}_{ij} = p\,\tilde{\Lambda}\,\tilde{g}_{ij}$$

Hence equations of motion:

$$f^2 a^{-2}\tilde{\Lambda} + \frac{a''}{a} - \frac{f'a'}{fa} + \frac{1}{2p}(\phi')^2 + \frac{1}{2p}(\chi')^2 = 0$$

$$p(p+1)f^2 a^{-2}\tilde{\Lambda} - p(p+1)\Big(\frac{a'}{a}\Big)^2 + \frac{1}{2}(\phi')^2 + \frac{1}{2}(\chi')^2 - \tfrac{1}{2}f^2 e^{b\phi+c\chi}Q^2 + n(n-1)f^2 e^{d\chi}\bar{K} = 0$$

$$\phi'' + (p+1)\frac{a'}{a}\phi' - \frac{f'}{f}\phi' - \tfrac{1}{2}b\,e^{b\phi+c\chi}f^2Q^2 = 0$$

$$\chi'' + (p+1)\frac{a'}{a}\chi' - \frac{f'}{f}\chi' - \tfrac{1}{2}c\,e^{b\phi+c\chi}f^2Q^2 + d\,n(n-1)e^{d\chi}f^2\bar{K} = 0$$

All know p-brane solutions recovered.

Example: M2-brane $\;(p = 2$, $n = 7$, $\tilde{\Lambda} = 0$, $\bar{K} = 1)$ is given by

$$a(r) \;=\; (r^6 + R_0^6)^{1/4}\, r^2 \qquad\qquad \chi(r) \;=\; \frac{1}{18\sqrt{7}}\,\ln(r^6 + R_0^6)$$

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since $(\alpha = \frac{\sqrt{7}}{6},\ \ \beta = -\frac{1}{3\sqrt{7}})$

$$\begin{aligned} \mathrm{d}\hat{s}^2 &= \mathrm{e}^{2\alpha\chi}\Big[a^2(z)\eta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + f^2(r)\mathrm{d}r^2\Big] + \mathrm{e}^{2\beta\chi}\,\mathrm{d}\Omega_7^2 \\ &= (r^6 + R_0^6)^{-7/6}\Big[(r^6 + R_0^6)^{1/2}\, r^4\, \eta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + (r^6 + R_0^6)^{3/2}\, r^{-2}\mathrm{d}r^2\Big] \\ &\qquad + (r^6 + R_0^6)^{1/3}\,\mathrm{d}\Omega_7^2 \\ &= \frac{r^4}{(r^6 + R_0^6)^{2/3}}\,\eta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + \frac{(r^6 + R_0^6)^{1/3}}{r^2}\Big[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_7^2\Big] \end{aligned}$$

3. The AJS domain wall

[Alonso-Alberca, B.J., Silva]

Single scalar solution (uncharged p-brane) of

$$S_{p+2} = \int \mathrm{d}^{p+2}x \sqrt{|g|}\Big[R \;-\; \tfrac{1}{2}(\partial\chi)^2 \;-\; \mathrm{e}^{d\chi}\Lambda\Big]$$

of the form

$$\Big[1 + d\sqrt{\tfrac{-\Lambda}{2p}}\; z\Big]^2 \tilde{g}_{ij}\, \mathrm{d}x^i \mathrm{d}x^j \;+\; \mathrm{d}z^2, \qquad\qquad \mathrm{e}^{\chi} \;=\; \Big[1 + d\sqrt{\tfrac{-\Lambda}{2p}}\; z\Big]^{-\frac{2}{d}}$$

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- in general: dS for $d^2 > 2/p$, AdS for $d^2 < 2/p$
- our case: $d = \sqrt{\frac{2(p+n)}{pn}}$, $\Lambda = -n(n-1)\bar{K}$
  - $\bar{K} = 1 \implies$ angular part is $\mathbb{S}^n$
  - $\tilde{\Lambda} = \frac{1}{3}(n-1) \implies$ de Sitter branes for $1 \leq p \leq D-4$

Our solution becomes:

$$ds^2 = \Big[1 + \lambda\, z\Big]^2 \tilde{g}_{ij}\, dx^i dx^j + dz^2 \qquad\qquad e^{\chi} = \Big[1 + \lambda\, z\Big]^{-\sqrt{\frac{2pn}{p+n}}}$$

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In D dimensions:

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$$(1 + \lambda\, z = r^{\lambda})$$

$$= r^{2\sqrt{\frac{n-1}{p+n}}} \tilde{g}_{ij}\, dx^i dx^j + r^{2\sqrt{\frac{n-1}{p+n}}-2} \Big[dr^2 + r^2 d\Omega_n^2\Big]$$

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Comment: this is scaling solution:

$a(z) = \Big(1 + \lambda\, z\Big)$ and $e^{\chi} = \Big[1 + \lambda\, z\Big]^{-\sqrt{\frac{2pn}{p+n}}}$ have same potentials in e.o.m.

$\longrightarrow$ use to construct two scalar solutions

4. Einstein branes with flux: general case

$$S = \int \mathrm{d}^{p+2}x \sqrt{|g|} \Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\chi)^2 - \tfrac{1}{2}\mathrm{e}^{b\phi + c\chi} Q^2 + n(n-1) e^{d\chi} \bar{K} \Big]$$

Scaling solution:

$$\mathrm{e}^{\phi} = a(z)^{N_1}, \qquad \mathrm{e}^{\chi} = a(z)^{N_2}.$$

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We find:

$$\mathrm{d}s^2 = \Big[1+\lambda\, z\Big]^2 \tilde{g}_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + \mathrm{d}z^2 \quad \mathrm{e}^{\phi} = \Big[1+\lambda\, z\Big]^{\frac{2p(n-1)}{(p+n)b}} \quad \mathrm{e}^{\chi} = \Big[1+\lambda\, z\Big]^{-\sqrt{\frac{2pn}{(p+n)}}}$$

with

$$\lambda = \frac{b(p+n)}{p}\sqrt{\frac{(n-1)\,\bar{K}}{(p+n)b^2+2(p+1)(n-1)}} \qquad Q^2 = \frac{4(n-1)^2(p+n)}{(p+n)b^2+2(p+1)(n-1)}$$

$$\tilde{\Lambda} = \frac{n-1}{p}\,\frac{(p+n)b^2-2(n-1)^2}{(p+n)b^2+2(p+1)(n-1)}\,\bar{K} \qquad \longrightarrow \bar{K} = 1$$

Uplifting:

$$d\hat{s}^2 = r^A\, \tilde{g}_{ij}\, dx^i dx^j \;+\; r^{A-2}\left[dr^2 + r^2 d\Omega_n^2\right],$$

$$e^{\hat{\phi}} = r^B, \qquad\qquad \hat{F}_{a_1\ldots a_n} = \frac{Q}{\sqrt{|\bar{h}|}}\,\bar{\varepsilon}_{a_1\ldots a_n}$$

with

$$A = A(p,n,b), \qquad B = B(p,n,b)$$

$$\tilde{R}_{ij} \;=\; -p\,\tilde{\Lambda}\,\tilde{g}_{ij}\ \ \text{(dS)} \qquad \text{for}\ \ b^2 > \tfrac{2(n-1)^2}{(p+n)}$$

$$\tilde{R}_{ij} \;=\; +p\,\tilde{\Lambda}\,\tilde{g}_{ij}\ \ \text{(AdS)} \qquad \text{for}\ \ b^2 < \tfrac{2(n-1)^2}{(p+n)}$$

$\longrightarrow$ strong dilaton coupling yields positive curvature branes
weak dilaton coupling yields negative curvature branes

5. Einstein branes in $D = 10$ supergravity $(p + n = 8)$

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$$A = \frac{1}{4}(p-3)\sqrt{\frac{7-p}{2}} \qquad B = \frac{1}{4}(7-p)\sqrt{2(7-p)} \qquad \tilde{\Lambda} = -\frac{(7-p)(5-p)}{2p}$$

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- $\tilde{\Lambda} < 0$ for $0 < p < 5$: AdS D-branes
- $\tilde{\Lambda} = 0$ for $p = 5$: Ricci flat D5-brane $\longrightarrow$ near-horizon of standard D5-brane
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- D7 and D8 are single scalar solutions (see previous section)
- D3 does not fit in out Ansatz $\Leftrightarrow$ no scaling solution

• Einstein F1 and NS5

F1 $(b = -1)$

$$\begin{cases} d\hat{s}^2 = r^{\frac{\sqrt{3}}{2}}\, \tilde{g}_{ij}\, dx^i dx^j \; + \; r^{\frac{\sqrt{3}}{2}-2} \Big[dr^2 + r^2 d\Omega_3^2 \Big] \\ e^{\hat{\phi}} = r^{3\sqrt{3}} \qquad\qquad \hat{F}_{a_1 \ldots a_3} = \frac{6}{\sqrt{|\bar{h}|}}\, \bar{\varepsilon}_{a_1 \ldots a_3} \\ \tilde{\Lambda} = -\frac{3}{4} \qquad\qquad \longrightarrow \text{AdS}_2 \end{cases}$$

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NS5 $(b=1)$

$$\begin{cases} d\hat{s}^2 = r^{\frac{1}{2}}\,\tilde{g}_{ij}\,dx^i dx^j \;+\; r^{\frac{11}{8}}\left[dr^2 + r^2 d\Omega_3^2\right] \\ e^{\hat{\phi}} = r^{8} \qquad \hat{F}_{a_1\ldots a_3} = \frac{2}{\sqrt{|\bar{h}|}}\,\bar{\varepsilon}_{a_1\ldots a_3} \\ \tilde{\Lambda} = 0 \qquad \longrightarrow \text{Ricci flat} \end{cases}$$

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$\longrightarrow$ S-dual to Einstein D1 and D5-branes

6. Scaling solutions and near horizon regions

DW / FLRW correspondence: relation between [Skenderis, Townsend]

$$ds^2 = -f^2(t)dt^2 + a^2(t)\tilde{g}_{ij}(x)\, dx^i dx^j$$

$$ds^2 = a^2(z)\tilde{g}_{ij}(x)\, dx^i dx^j + f^2(z)dz^2$$

provided $t \leftrightarrow z$ $\tilde{\Lambda} \leftrightarrow -\tilde{\Lambda}$ $V(\phi) \leftrightarrow -V(\phi)$

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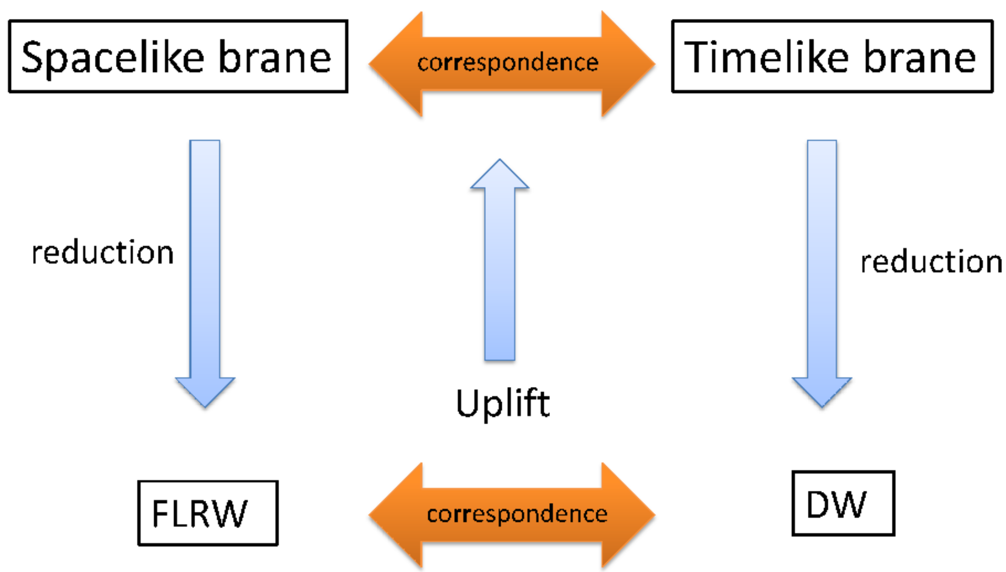

$\longrightarrow$ generalisable to timelike / spacelike brane correspondence

- Cosmological models (hence DW models) with exponential potentials

$$S = \int \mathrm{d}^{p+2}x\sqrt{|g|}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\chi)^2 - \tfrac{1}{2}\mathrm{e}^{b\phi+c\chi}Q^2 + n(n-1)e^{d\chi}\bar{K}\Big]$$

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$\longrightarrow$ flat DW: $a(z) \sim z^{\ell}$ with $\ell = \frac{(9-p)}{(p-3)^2}$

Curved DW: $a(z) \sim z$

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- cases without curved scaling solution: (AdS with $Q = 0$, D3-brane, ...)

$\longrightarrow$ solutions should exist

just do not have scaling regime

7. Conclusions

- Constructed domain wall solutions with Einstein worldvolume

$$ds^2 = z^\ell \tilde{g}_{ij} dx^i dx^j + dz^2 \qquad\qquad \tilde{R}_{ij} = p\,\tilde{\Lambda}\,\tilde{g}_{ij}$$

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- Exact solutions are scaling solutions and hence near horizon limits
  Full solutions to be determined

Future work:

- Look for full solutions

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Thank you!

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