Einstein branes

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References: JHEP 10 (2011) 002; ArXiv:1107.1427;

Aim:

Curved p-brane solutions with Einstein geometry in worldvolume

$$d\hat{s}^2 = H^A(r)\, \tilde{g}_{ij}(x)\, dx^i dx^j \ + \ H^B(r) \Big[dr^2 + r^2 d\Omega_n^2 \Big]$$

$$\text{with} \qquad \tilde{R}_{ij}(x) = p\, \tilde{\Lambda}\, \tilde{g}_{ij}(x)$$

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  $\longrightarrow$ Randall-Sundrum type of analysis

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- Alternative way of finding de Sitter solutions in string theory
  $\longrightarrow$ Randall-Sundrum type of analysis
- Flux compactificaction yield curved p-brane solutions
  $\longrightarrow$ backreaction poorly studied
- AdS domain walls sometimes supersymmetric. AdS -branes also?
  $\longrightarrow$ new class of supersymmetric solutions for AdS-branes?

Outlook

1. Introduction: flat p-branes and their curved generalizations

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6. Conclusions

1. Introduction

The importance of p-branes in string theory can hardly be overestimated:

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- solitonic solutions in supergravity

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The importance of p-branes in string theory can hardly be overestimated:

- fundamental objects in string theory
- solitonic solutions in supergravity
- crucial for understanding of
  - gauge/gravity duals
  - black hole entropy
  - string phenomenology & cosmology
  - non-perturbative effects
  - ...

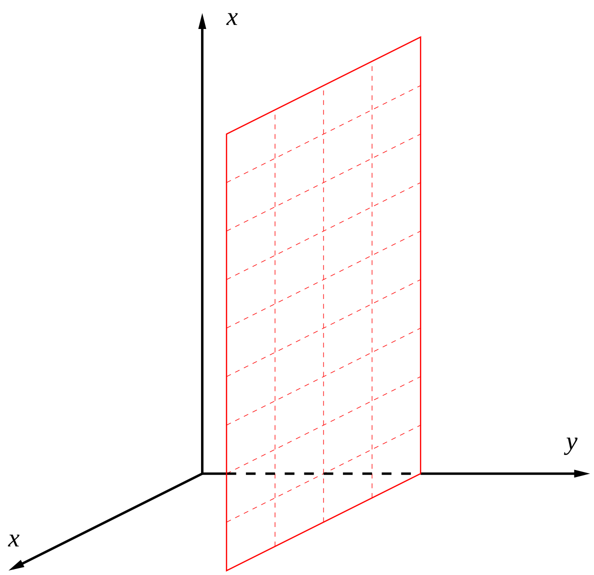
In supergravity: solutions of

$$S_D = \int \mathrm{d}^D x \sqrt{|\hat{g}|} \left[\hat{R} - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2(p+2)!} \mathrm{e}^{b\hat{\phi}} \hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_{p+2}} \hat{F}^{\hat{\mu}_1 \dots \hat{\mu}_{p+2}} \right]$$

of the form

$$\mathrm{d}\hat{s}^2 = H^{2A}(r)\, \eta_{ij}\, \mathrm{d}x^i \mathrm{d}x^j + H^{2B}(r) \Big(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_n^2 \Big),$$

$$e^{-2\hat{\phi}} = H^C(r), \qquad \hat{F}_{i_1 \dots i_{p+1} r} = \partial_r H^E(r)\, \varepsilon_{i_1 \dots i_{p+1}}$$



- Planar extended objects with p spatial directions: $SO(p,1) \times SO(n)$ symm
- Electrically and/or magnetically charged under $\hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_{p+2}}$
- $D = 11,\ p = 2$: M2 and M5-brane
  $D = 10,\ p = 1$: F1 and NS5-brane
  $D = 10,\ 0 \leq p \leq 8$: Dp-branes

[Duff et al.] [Güven] [Dahbolkar et al.] [Callan, et al.] [Duff et al.] [Polchinski]

General solution:

$$\mathrm{d}\hat{s}^2 \;=\; H^{2A}(r)\,\eta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j \;+\; H^{2B}(r)\Big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_n^2\Big),$$

$$e^{-2\hat{\phi}} \;=\; H^C(r), \qquad\qquad \hat{F}_{i_1\ldots i_{p+1}r} \;=\; \partial_r H^{-1}(r)\,\varepsilon_{i_1\ldots i_{p+1}}$$

with

$$A = -\frac{4(D-p-3)}{\Delta(D-2)} \qquad B = \frac{4(p+1)}{\Delta(D-2)} \qquad C = \frac{4b}{\Delta}$$

$$\Delta = b^2 + 2\frac{(p+1)(D-p-3)}{D-2} \qquad\qquad \nabla^2 H(r) = 0$$

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Known generalisations:

- Ricci-flat branes [Brecher et al.] [B.J.] [Figueroa-O'Farrill]
- Curved domain walls [Kachru et al.] [Alonso-Alberca et al.] ...

Ricci-flat branes:

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still solution if

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$$\Delta = b^2 + 2\frac{(p+1)(D-p-3)}{D-2} \qquad \nabla^2 H(r) = 0 \qquad \tilde{R}_{ij} = 0$$

Reason: factorisation of x-dependence

$$R_{ij} = \tilde{R}_{ij} \;-\; \tilde{g}_{ij} H^{2(A-B)-1}\Big[\nabla^2 H + (\partial H)^2\Big]$$

$\longrightarrow$ eqn of motions:

$$\tilde{R}_{ij} \;-\; \tilde{g}_{ij} H^{2(A-B)-1}\Big[\nabla^2 H + (\partial H)^2\Big] + \tilde{g}_{ij}(\partial\hat{\phi})^2 + \tilde{g}_{ij}\, e^{b\hat{\phi}}\,\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\hat{F}^{\hat{\mu}_1\ldots\hat{\mu}_{p+2}} = 0$$

Curved domain walls (exact and/or numerical):

Solutions of

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of the form

$$\mathrm{d}\hat{s}^2 = H^{2A}(r)\, \tilde{g}_{ij}(x)\, \mathrm{d}x^i \mathrm{d}x^j + H^{2B}(r) \mathrm{d}r^2$$

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No known solutions of general p-branes with $\tilde{R}_{ij} = p\tilde{\Lambda}\, \tilde{g}_{ij}$ (Einstein branes)

$\longrightarrow$ Knippenberg (2003): such solutions do not exist

End of the story (?)

One to one correspondence between p-brane and DW

$$\mathrm{d}\hat{s}^2 = \mathrm{e}^{2A(r)}\,\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j + \mathrm{e}^{2B(r)}\Big(\mathrm{d}r^2 + r^2\mathrm{d}\Sigma_n^2\Big)$$

$$\mathrm{d}s^2 = a^2(z)\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j + f^2(z)\mathrm{d}z^2$$

through dimensional reduction over angular part $\mathrm{d}\Sigma_n^2 = \bar{h}_{ab}(\theta)\mathrm{d}\theta^a\mathrm{d}\theta^b$ with

$$\bar{R}_{ab} = (n-1)\bar{K}\bar{h}_{ab}, \qquad\qquad \bar{K} = 0, \pm 1$$

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Strategy:

1. Look for solutions

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2. Uplift solution to

$$d\hat{s}^2 = e^{2A(r)}\,\tilde{g}_{ij}\,dx^i dx^j + e^{2B(r)}\Big(dr^2 + r^2 d\Sigma_n^2\Big) \qquad \text{with } \tilde{R}_{ij} = p\tilde{\Lambda}\,\tilde{g}_{ij}$$

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$\longrightarrow$ Knippenberg's argument invalidated: too restrictive Ansatz?

2. Reduction to domain wall problem

Magnetic Ansatz:

$$d\hat{s}^2 \;=\; e^{2\alpha\chi}\, g_{\mu\nu}\, dx^\mu dx^\nu \;+\; e^{2\beta\chi}\, \bar{h}_{ab}\, d\theta^a d\theta^b$$

$$\hat{\phi} = \phi(x), \qquad\qquad \hat{F}_{a_1\ldots a_n} \;=\; \frac{1}{\sqrt{|\bar{h}|}}\, Q\, \bar{\varepsilon}_{a_1\ldots a_n}$$

with:

- $\mu, \nu \in \{0, \ldots, p+2\}, \quad a, b \in \{1, \ldots, n\}$
- $\bar{h}_{ab}(\theta)$ is metric of angular part of transverse space with $\bar{R}_{ab} = (n-1)\bar{K}\,\bar{h}_{ab}$
- $\chi(x)$ is breathing mode
- $g_{\mu\nu}(x)$ is metric of domain wall spacetime

$$ds^2 \;=\; g_{\mu\nu}\, dx^\mu dx^\nu \;=\; a^2(z)\tilde{g}_{ij}(x)\, dx^i dx^j \;+\; f^2(z) dz^2$$

Dimensional reduction: $(\tilde{D} = p\alpha + n\beta)$

$$\hat{R} \;=\; \mathrm{e}^{-2\alpha\chi} R \;+\; n(n-1)\mathrm{e}^{-2\beta\chi}\bar{K}$$
$$+\; 2(\tilde{D}+\alpha)\mathrm{e}^{-2\alpha\chi}\nabla^2\chi \;+\; \Big[\tilde{D}^2 + p\alpha^2 + n\beta^2\Big]\mathrm{e}^{-2\alpha\chi}(\partial\chi)^2$$

hence

$$S \;=\; \int \mathrm{d}^D x\sqrt{|\hat{g}|}\,\Big[\hat{R} \;-\; \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2(p+2)!}\,\mathrm{e}^{b\hat{\phi}}\hat{F}_{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\hat{F}^{\hat{\mu}_1\ldots\hat{\mu}_{p+2}}\Big]$$
$$=\; \int \mathrm{d}^{p+2}x\sqrt{|g|}\Big[R \;-\; \tfrac{1}{2}(\partial\phi)^2 \;-\; \tfrac{1}{2}(\partial\chi)^2 \;-\; \tfrac{1}{2}\mathrm{e}^{b\phi+c\chi}Q^2 \;+\; n(n-1)e^{d\chi}\bar{K}\Big]$$

provided that $\quad p\alpha = -n\beta \quad$ and $\quad \alpha = \sqrt{\frac{n}{2p(p+n)}}$

$$\Longrightarrow \quad \beta = \beta(p,n), \qquad c = c(p,n), \qquad d = d(p,n)$$

$\longrightarrow$ gravity coupled to 2 scalars in double exponential potential

Curved domain wall Ansatz:

$$\mathrm{d}s^2 \;=\; a^2(z)\tilde{g}_{ij}(x)\,\mathrm{d}x^i\mathrm{d}x^j \;+\; f^2(z)\mathrm{d}z^2$$

$$\phi = \phi(z), \qquad \chi = \chi(z), \qquad \tilde{R}_{ij} = p\,\tilde{\Lambda}\,\tilde{g}_{ij}$$

Hence equations of motion:

$$f^2 a^{-2}\tilde{\Lambda} \;+\; \frac{a''}{a} \;-\; \frac{f'a'}{fa} \;+\; \frac{1}{2p}(\phi')^2 \;+\; \frac{1}{2p}(\chi')^2 \;=\; 0$$

$$p(p+1)f^2a^{-2}\tilde{\Lambda} \;-\; p(p+1)\Big(\frac{a'}{a}\Big)^2 \;+\; \frac{1}{2}(\phi')^2 \;+\; \frac{1}{2}(\chi')^2$$
$$-\;\tfrac{1}{2}f^2\mathrm{e}^{b\phi+c\chi}Q^2 \;+\; n(n-1)f^2\mathrm{e}^{d\chi}\bar{K} \;=\; 0$$

$$\phi'' \;+\; (p+1)\,\frac{a'}{a}\,\phi' \;-\; \frac{f'}{f}\,\phi' \;-\; \tfrac{1}{2}b\,\mathrm{e}^{b\phi+c\chi}f^2Q^2 \;=\; 0$$

$$\chi'' \;+\; (p+1)\,\frac{a'}{a}\,\chi' \;-\; \frac{f'}{f}\,\chi' \;-\; \tfrac{1}{2}c\,\mathrm{e}^{b\phi+c\chi}f^2Q^2 \;+\; d\,n(n-1)\mathrm{e}^{d\chi}f^2\bar{K} \;=\; 0$$

$\longrightarrow$ All know p-brane solutions recovered.

3. Einstein branes with flux: general case

$$S = \int \mathrm{d}^{p+2}x \sqrt{|g|} \Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\chi)^2 - \tfrac{1}{2}\mathrm{e}^{b\phi + c\chi} Q^2 + n(n-1) e^{d\chi} \bar{K} \Big]$$

Scaling solution:

$$\mathrm{e}^{\phi} = a(z)^{N_1}, \qquad \mathrm{e}^{\chi} = a(z)^{N_2}.$$

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We find:

$$\mathrm{d}s^2 = \Big[1+\lambda\, z\Big]^2 \tilde{g}_{ij}\,\mathrm{d}x^i\mathrm{d}x^j + \mathrm{d}z^2 \qquad \mathrm{e}^{\phi} = \Big[1+\lambda\, z\Big]^{\frac{2p(n-1)}{(p+n)b}} \qquad \mathrm{e}^{\chi} = \Big[1+\lambda\, z\Big]^{-\sqrt{\frac{2pn}{(p+n)}}}$$

with

$$\lambda = \frac{b(p+n)}{p}\sqrt{\frac{(n-1)\,\bar{K}}{(p+n)b^2+2(p+1)(n-1)}} \qquad Q^2 = \frac{4(n-1)^2(p+n)}{(p+n)b^2+2(p+1)(n-1)}$$

$$\tilde{\Lambda} = \frac{n-1}{p}\,\frac{(p+n)b^2-2(n-1)^2}{(p+n)b^2+2(p+1)(n-1)}\,\bar{K} \qquad\qquad \longrightarrow \bar{K}=1$$

Uplifting:

$$d\hat{s}^2 = r^A\,\tilde{g}_{ij}\,dx^i dx^j \;+\; r^{A-2}\left[dr^2 + r^2 d\Omega_n^2\right],$$

$$e^{\hat{\phi}} = r^B, \qquad\qquad \hat{F}_{a_1\ldots a_n} = \frac{Q}{\sqrt{|\bar{h}|}}\,\bar{\varepsilon}_{a_1\ldots a_n}$$

with

$$A = A(p,n,b), \qquad B = B(p,n,b)$$

$$\tilde{R}_{ij} \;=\; -p\,\tilde{\Lambda}\,\tilde{g}_{ij}\ \text{(dS)} \qquad \text{for}\ b^2 > \tfrac{2(n-1)^2}{(p+n)}$$

$$\tilde{R}_{ij} \;=\; +p\,\tilde{\Lambda}\,\tilde{g}_{ij}\ \text{(AdS)} \qquad \text{for}\ b^2 < \tfrac{2(n-1)^2}{(p+n)}$$

$\longrightarrow$ strong dilaton coupling yields positive curvature branes
weak dilaton coupling yields negative curvature branes

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with

$$A = \frac{1}{4}(p-3)\sqrt{\frac{7-p}{2}} \qquad B = \frac{1}{4}(7-p)\sqrt{2(7-p)} \qquad \tilde{\Lambda} = -\frac{(7-p)(5-p)}{2p}$$

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- $\tilde{\Lambda} < 0$ for $1 \leq p \leq 4$: AdS D-branes
- $\tilde{\Lambda} = 0$ for $p = 5$: Ricci flat D5-brane $\longrightarrow$ near-horizon of standard D5-brane
- $\tilde{\Lambda} > 0$ for $p = 6$: dS D6-branes

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- D7 and D8 are single scalar solutions (see previous section)
- D3 does not fit in out Ansatz $\Leftrightarrow$ no scaling solution

• Einstein F1 and NS5

F1 $(b=-1)$

$$\begin{cases} d\hat{s}^2 = r^{\frac{\sqrt{3}}{2}}\,\tilde{g}_{ij}\,dx^i dx^j \;+\; r^{\frac{\sqrt{3}}{2}-2}\left[dr^2 + r^2 d\Omega_3^2\right] \\ e^{\hat{\phi}} = r^{3\sqrt{3}} \qquad\qquad \hat{F}_{a_1\ldots a_3} = \frac{6}{\sqrt{|\bar{h}|}}\,\bar{\varepsilon}_{a_1\ldots a_3} \\ \tilde{\Lambda} = -\frac{3}{4} \qquad\qquad \longrightarrow \mathrm{AdS}_2 \end{cases}$$

• Einstein F1 and NS5

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$\longrightarrow$ S-dual to Einstein D1 and D5-branes

6. Scaling solutions and near horizon regions

DW / FLRW correspondence: relation between [Skenderis, Townsend]

$$ds^2 = -f^2(t)dt^2 + a^2(t)\tilde{g}_{ij}(x)\,dx^i dx^j$$

$$ds^2 = a^2(z)\tilde{g}_{ij}(x)\,dx^i dx^j + f^2(z)dz^2$$

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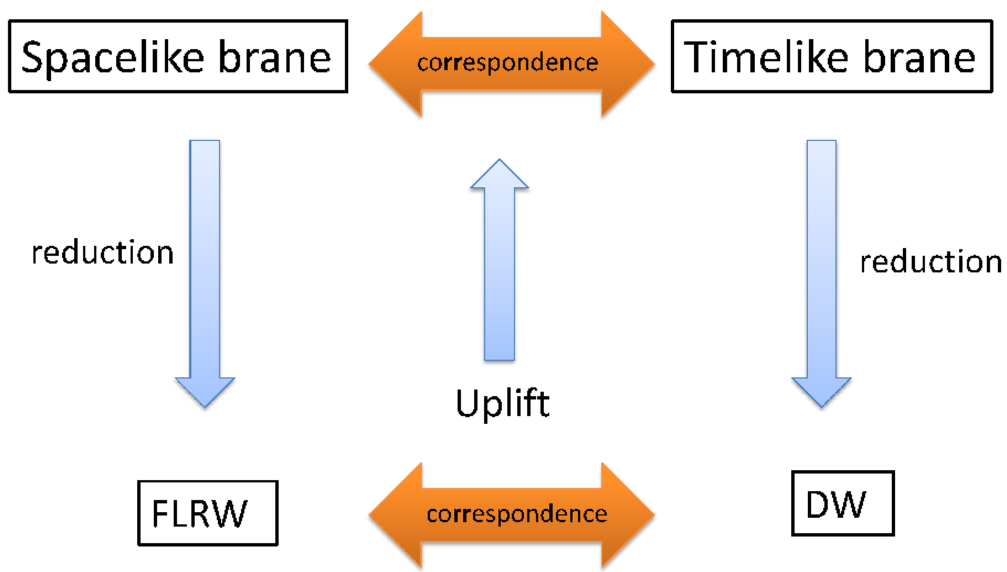

$\longrightarrow$ generalisable to timelike / spacelike brane correspondence

- Cosmological models (hence DW models) with exponential potentials

$$S = \int \mathrm{d}^{p+2}x\sqrt{|g|}\Big[R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\chi)^2 - \tfrac{1}{2}\mathrm{e}^{b\phi+c\chi}Q^2 + n(n-1)e^{d\chi}\bar{K}\Big]$$

form autonomous system: flow equations in flow diagram

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$\longrightarrow$ flat DW: $a(z) \sim z^{\ell}$ with $\ell = \frac{(9-p)}{(p-3)^2}$

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- cases without curved scaling solution: (AdS with $Q = 0$, D3-brane, ...)

$\longrightarrow$ solutions should exist

just do not have scaling regime

7. Conclusions

- Constructed domain wall solutions with Einstein worldvolume

$$ds^2 = z^\ell \tilde{g}_{ij} dx^i dx^j + dz^2 \qquad\qquad \tilde{R}_{ij} = p\, \tilde{\Lambda}\, \tilde{g}_{ij}$$

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- Exact solutions are scaling solutions and hence near horizon limits
  Full solutions to be determined

Future work:

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Thank you!

special thanks to:

Dr. Thomas S. Harvey with Einstein's brain