Backreaction in warped compatifications of supersymmetric domain walls

Bert Janssen

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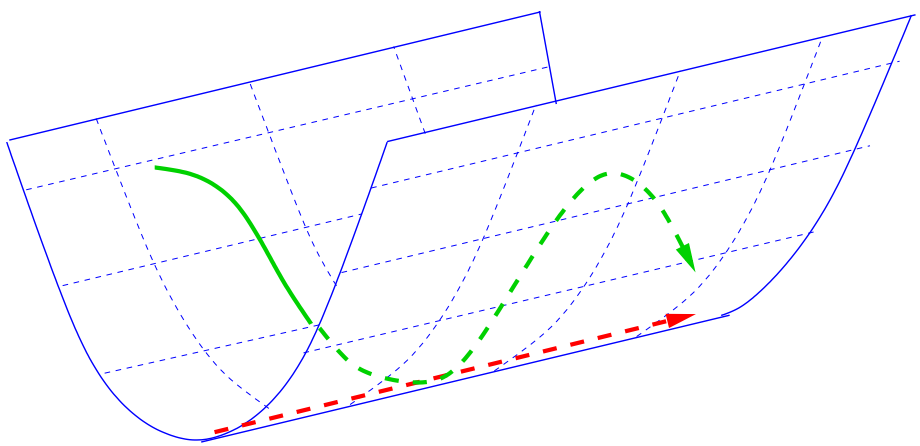

In collaboration with: J. Blåbäck, B. Vercnocke and T. Van Riet

References: JHEP 10 (2012) 139 (arXiv:1207.0814) and arXiv: 1312.6125 (to appear in JHEP).

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1. Extra dimensions: what and why

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2. Compactification in supergravity

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3. p-brane solutions

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5. General backreacted domain walls

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1. Extra dimensions: what and why
2. Compactification in supergravity
3. p-brane solutions
4. Fractional dynamical branes
5. General backreacted domain walls
6. Conclusions

1. Extra dimensions: What and why

Gravity is poorly understood at small scales:

- Theoretically: General relativity is not renormalisable
  How real are singularities?
  Few clues about quantum gravity
  ...

1. Extra dimensions: What and why

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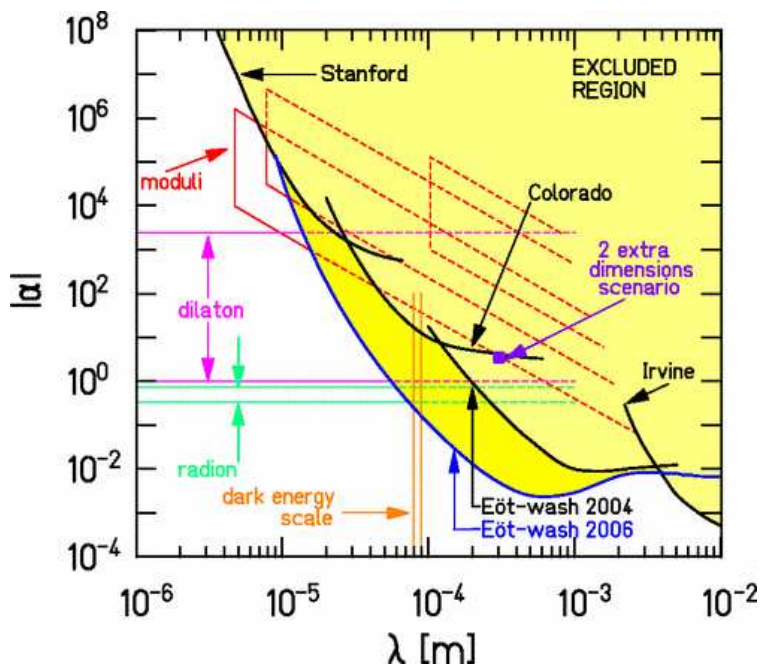
- Experimentally: Hard to test at small scales

$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 + \alpha\, e^{-r/\lambda}\right]$$

$$|\alpha| = 1 \implies \lambda \leq 56\,\mu m$$

$$|\alpha| = 8/3 \implies \lambda \leq 44\,\mu m$$

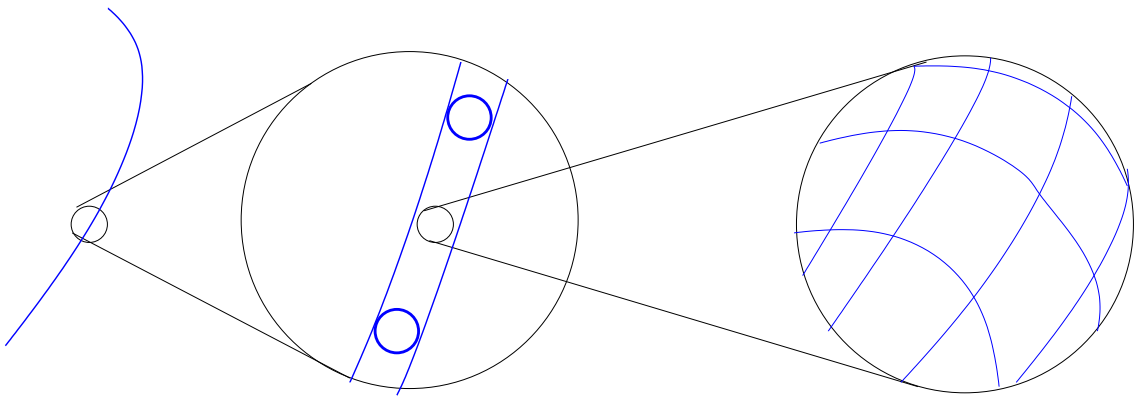


[Kapner et al., 2007]

Dimensional reduction

Kaluza (1921): Unification of General Relativity and Maxwell theory (with scalar) in D=5 in presence of isometry

Klein (1926): Fifth dimension is compact and small



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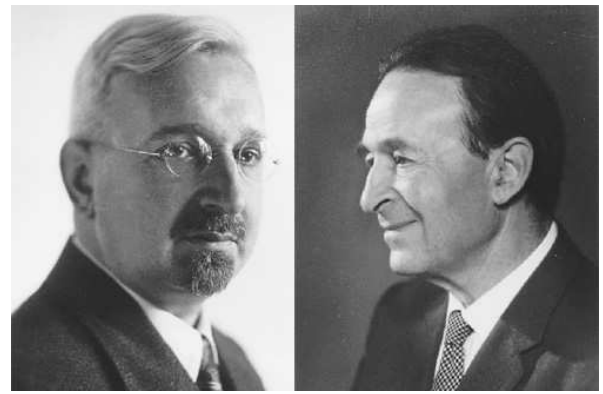

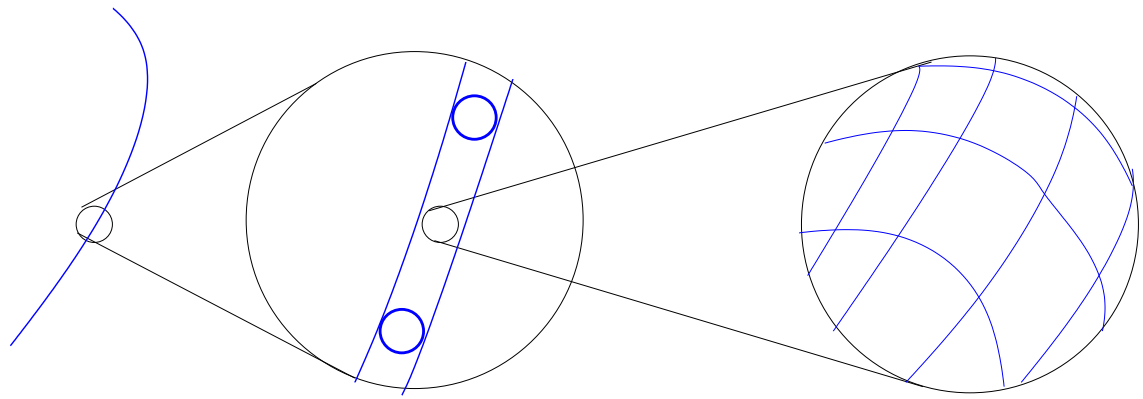

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$$\partial_{\hat{\mu}} \partial^{\hat{\mu}} \hat{\phi} \;=\; \sum_n \Big(\partial_\mu \partial^\mu \phi_n \;+\; m_n^2 \phi_n \Big) = 0 \qquad \text{con} \quad m_n = \frac{n}{R}$$

5-dimensional pure gravity:

$$\hat{g}_{\hat{\mu}\hat{\nu}} \longrightarrow g_{\mu\nu} + A_\mu + k$$

$$\mathbf{15} \longrightarrow \mathbf{10} + \mathbf{4} + \mathbf{1}$$

$$g_{\mu\nu} = \hat{g}_{\mu\nu} - \frac{\hat{g}_{\mu z}\hat{g}_{\nu z}}{\hat{g}_{zz}}, \qquad A_\mu = \frac{\hat{g}_{\mu z}}{\hat{g}_{zz}}, \qquad k = \hat{g}_{zz}$$

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reparametrisation $z \longrightarrow z + \xi(x) \sim U(1)$ gauge transformation

$$\delta_5\, g_{\mu\nu} = \delta_4\, g_{\mu\nu}, \qquad \delta_5\, A_\mu = \delta_4\, A_\mu + \partial_\mu \xi, \qquad \delta_5\, k = \delta_4\, k$$

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Einstein-Hilbert $\longrightarrow$ Einstein-Maxwell-dilaton

$$\begin{aligned} S &= \frac{1}{\kappa_5} \int d^5x \sqrt{|\hat{g}|}\, \hat{R} \\ &= \frac{1}{\kappa_4} \int d^4x \sqrt{|g|} k \Big[R + (\partial \log k)^2 - k^2 F_{\mu\nu} F^{\mu\nu} \Big]. \end{aligned}$$

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  $\longrightarrow$ theory ignored untill arrival of supergravity and string theory, who live naturally in 10 and 11 dimensions

  $\longrightarrow$ dimensional reduction necessary for realistic theory

2. Compactification in supergravity

Supergravity = Einstein-gravity coupled to bosonic and fermionic fields , invariant under local supersymmetry transformations:

$$S = \frac{1}{2\kappa} \int d^D x \sqrt{|g|} \Big[R \; + \; \tfrac{1}{2}(\partial\phi)^2 \; + \; \tfrac{1}{12} e^{\phi} H_{\mu\nu\rho} H^{\mu\nu\rho} \; + \ldots \Big]$$

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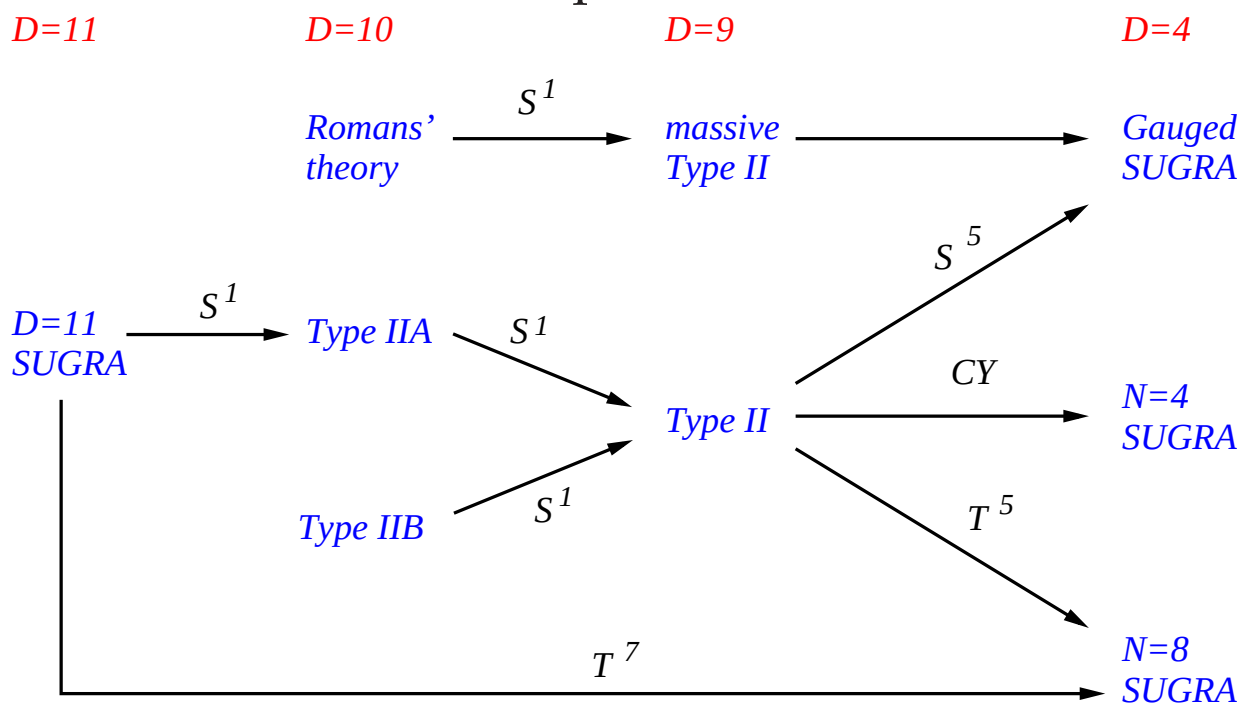
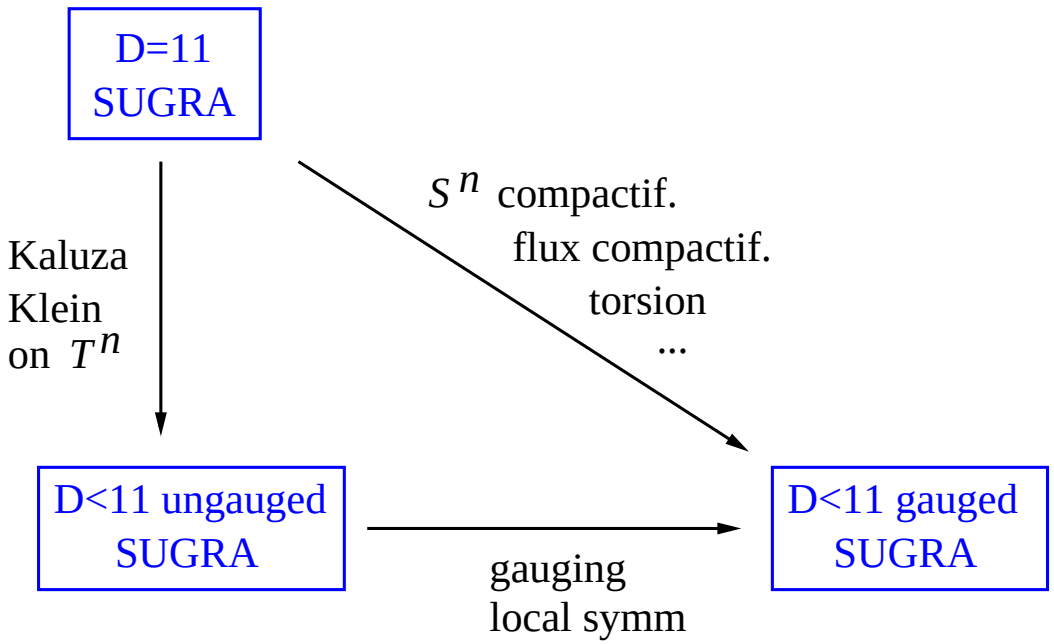
- Appear as low-energy limits of string theory
- Precise content and interactions depend on dimensions and supersymmetry



Gauged Supergravities:



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- Large global symm groups
- running moduli
- Max supersymm Minkowski vacuum

- Less than maximal susy
- Non-Abelian gauge groups
- Scalar potential
- No Minkowski vacuum

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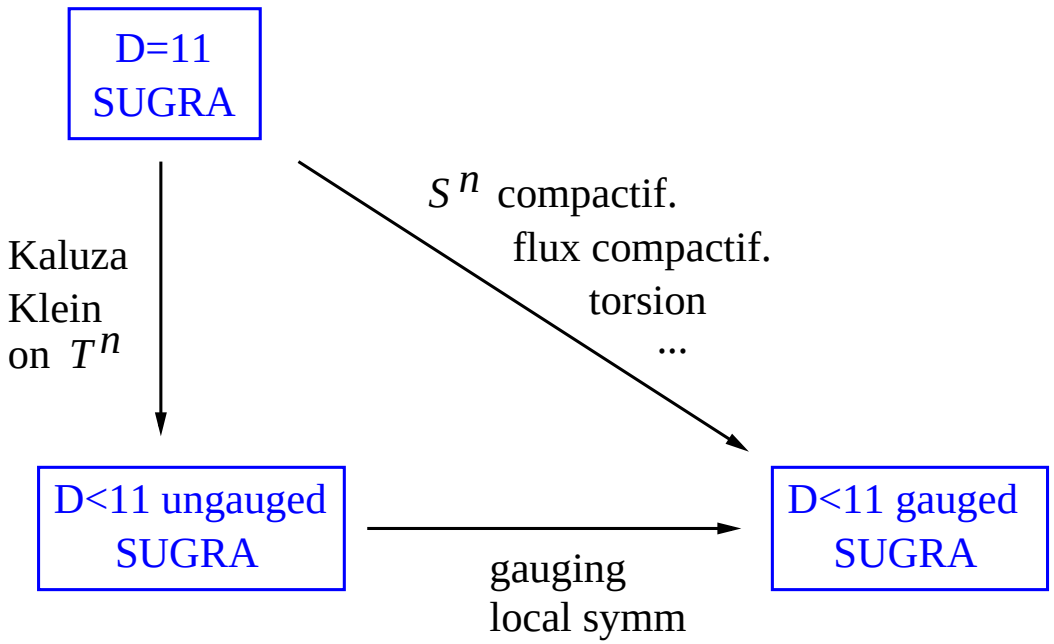

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E.g.: 5-dim maximally $SO(6)$ gauge supergravity (gravity side of AdS/CFT)
$\longrightarrow$ 10-dim Type IIB compactified on $AdS_5 \times S^5$

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BUT: No-Go theorem

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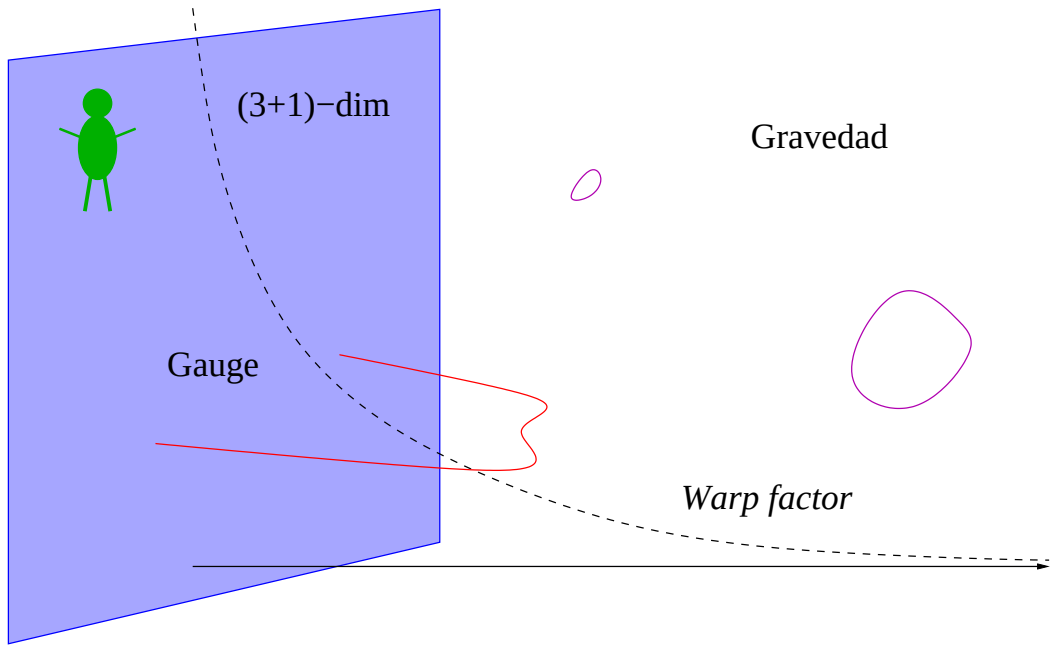
$\longrightarrow$ Orientifolds: Negative tension objects, that projects out odd part of field content

Orientifolds and warped compactifications:

3-brane in AdS_5: $ds^2 = e^{2|z|/R_0}\eta^{\mu\nu}dx^\mu dx^\nu - dz^2$ [Randall, Sundrum, 1999]



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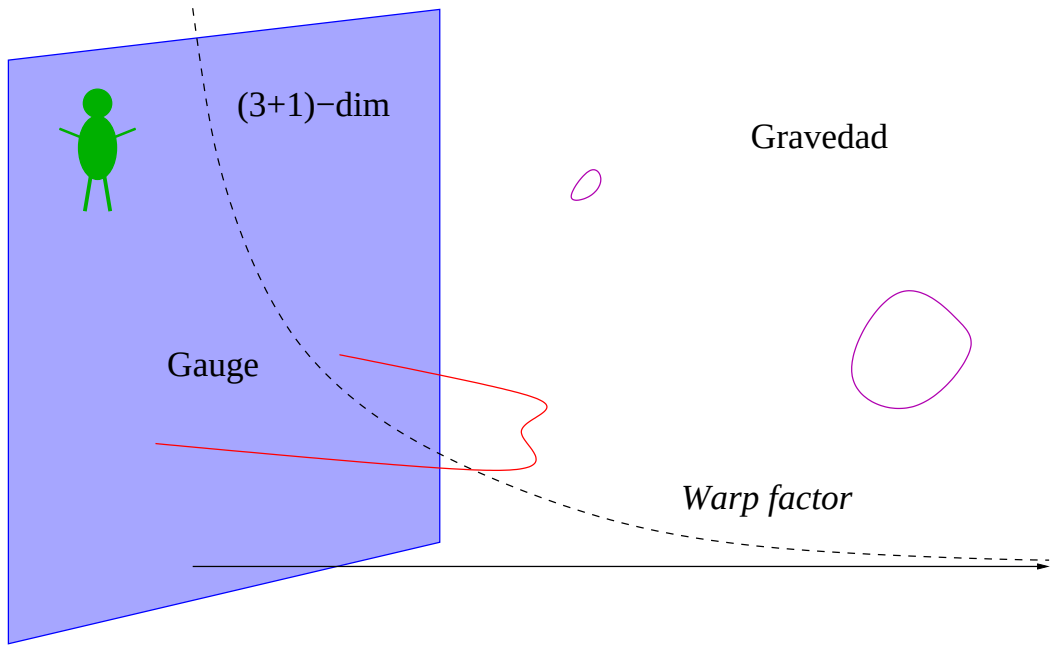
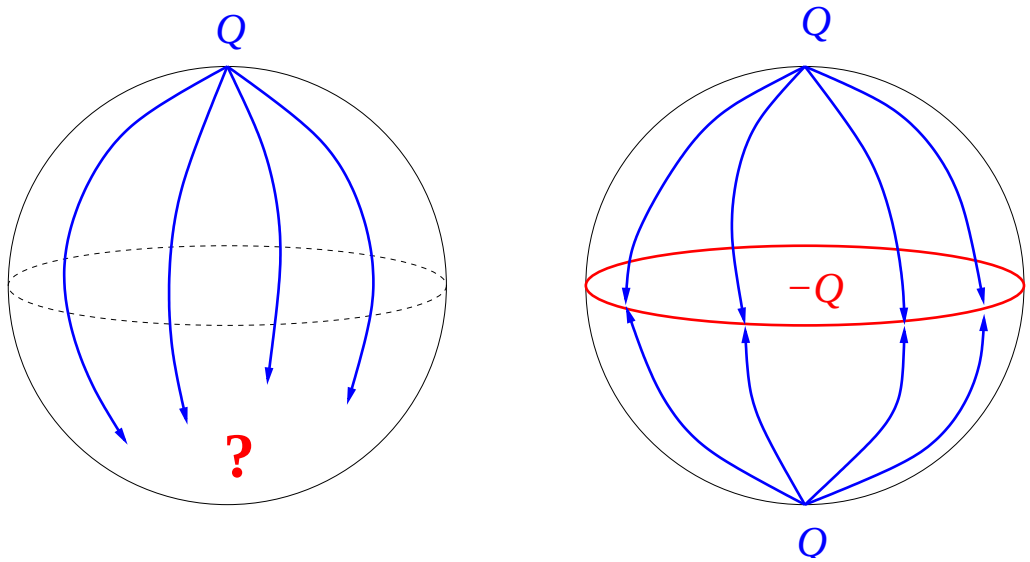


- $z \to -z$ symmetry proyects out odd modes
- Used to cancel tadpole conditions:

Tadpole cancelation: charge on compact manifold



$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} + m B_{\mu\nu} \implies dF_2 = m H_3 + \rho$$

$$\iff 0 = m \int H_3 + Q$$

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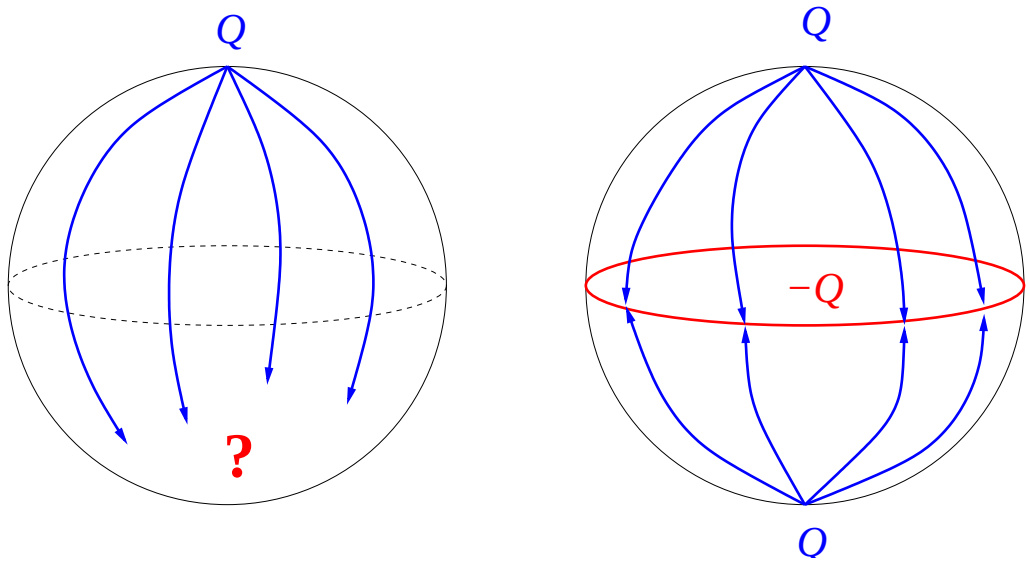

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Electromagnetic attraction cancels gravitational repulsion

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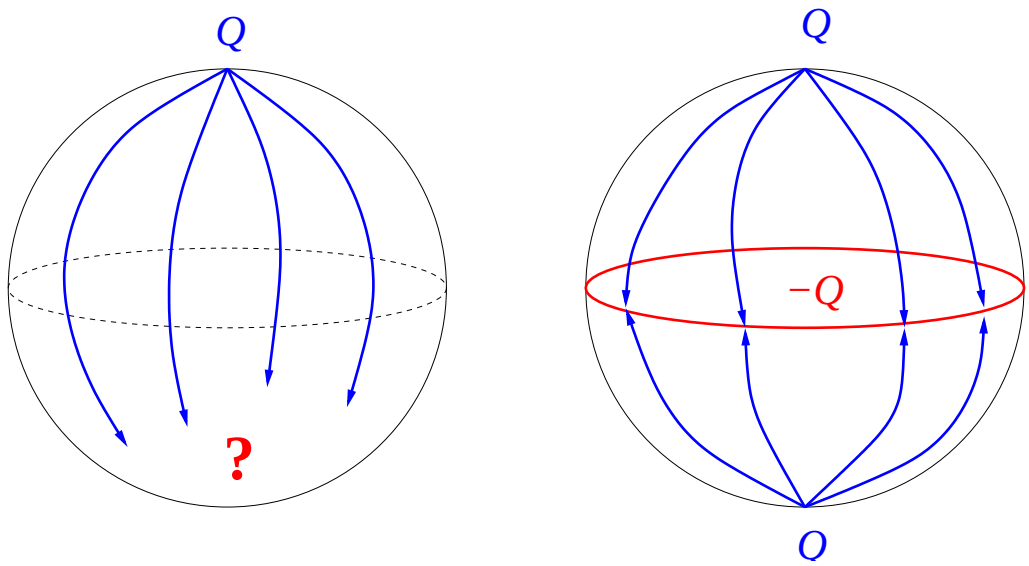

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BPS condition (no-force condition) of BPS orientifolds:

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BPS condition (no-force condition) of extremal black holes and p-branes:

Electromagnetic repulsion cancels gravitational attraction

$\longrightarrow$ similarity between orientifolds and p-branes!

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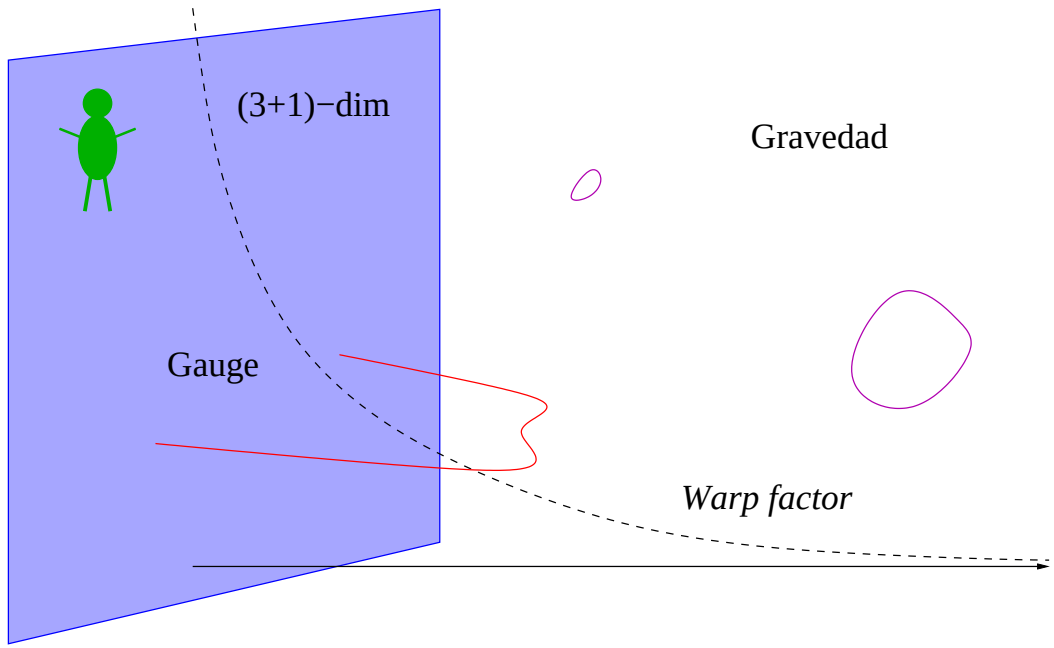

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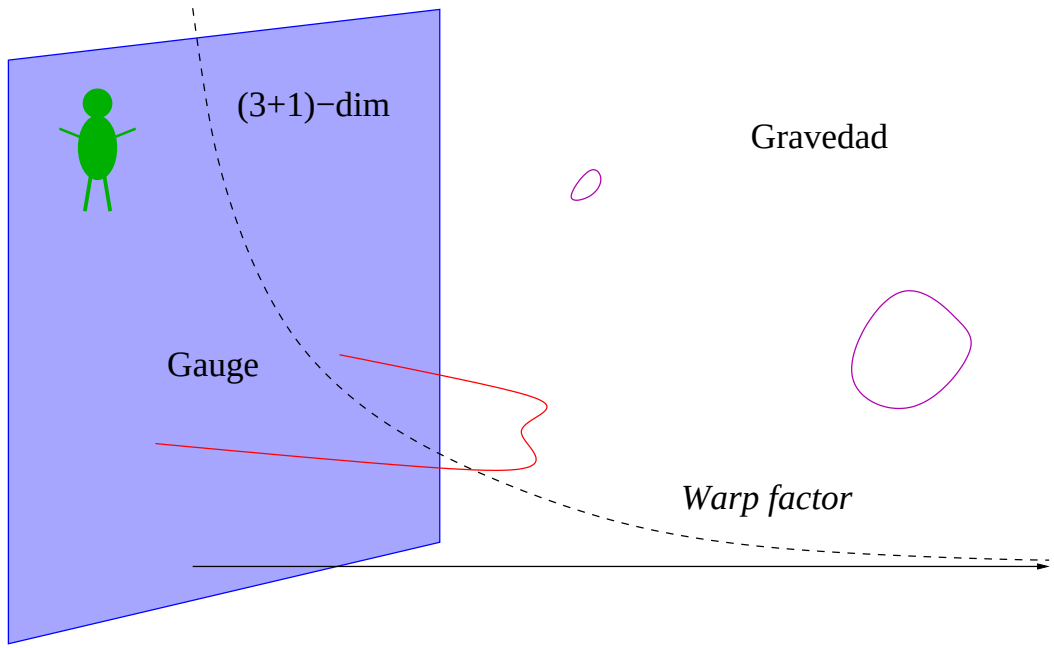

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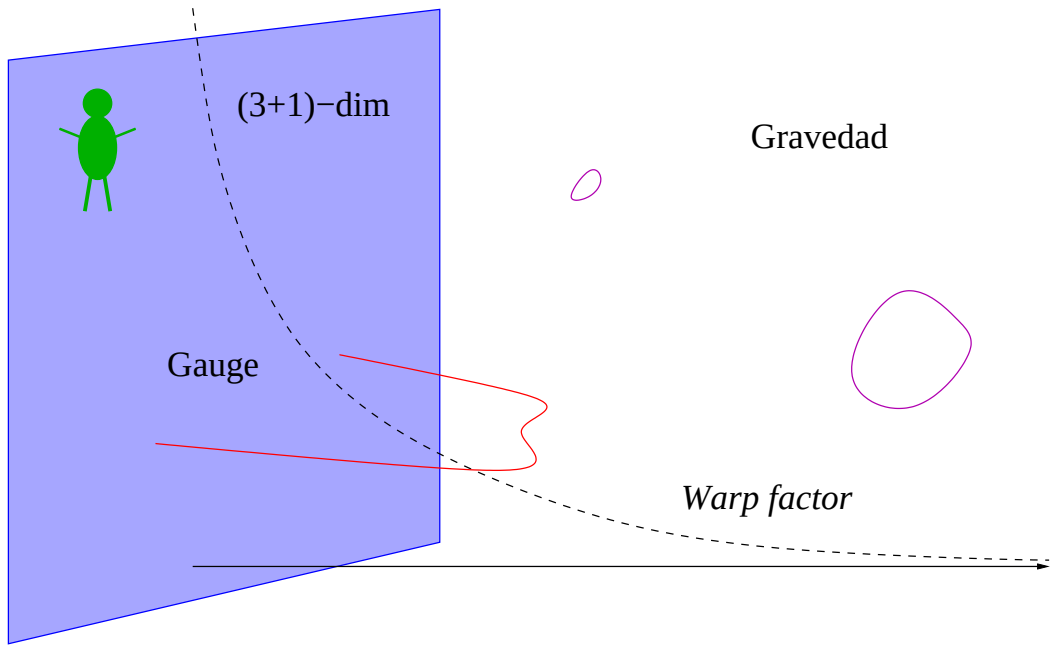

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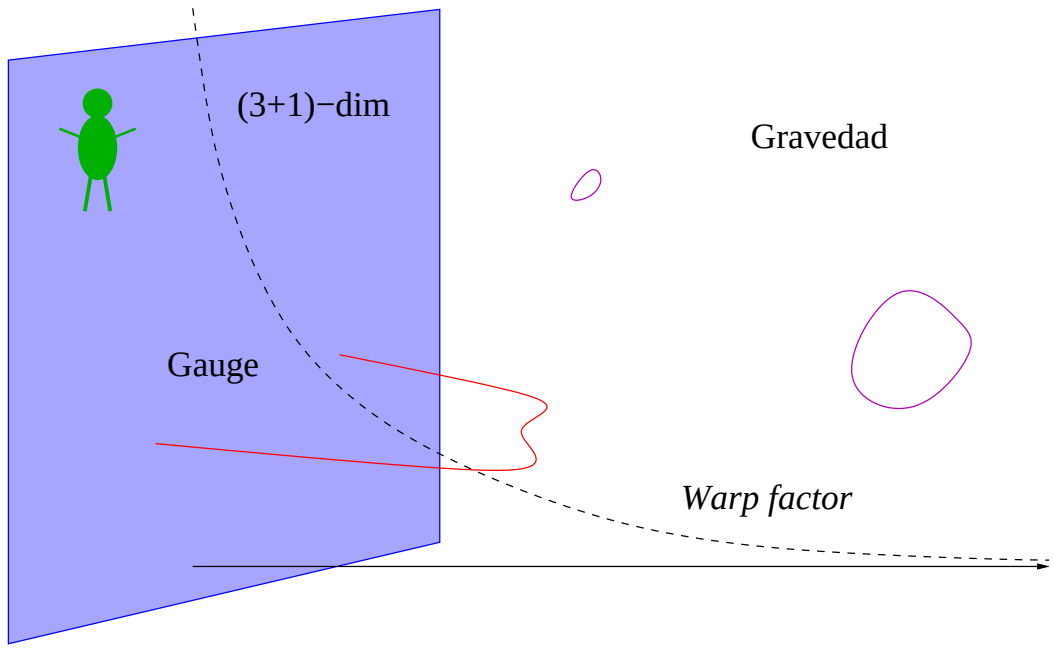

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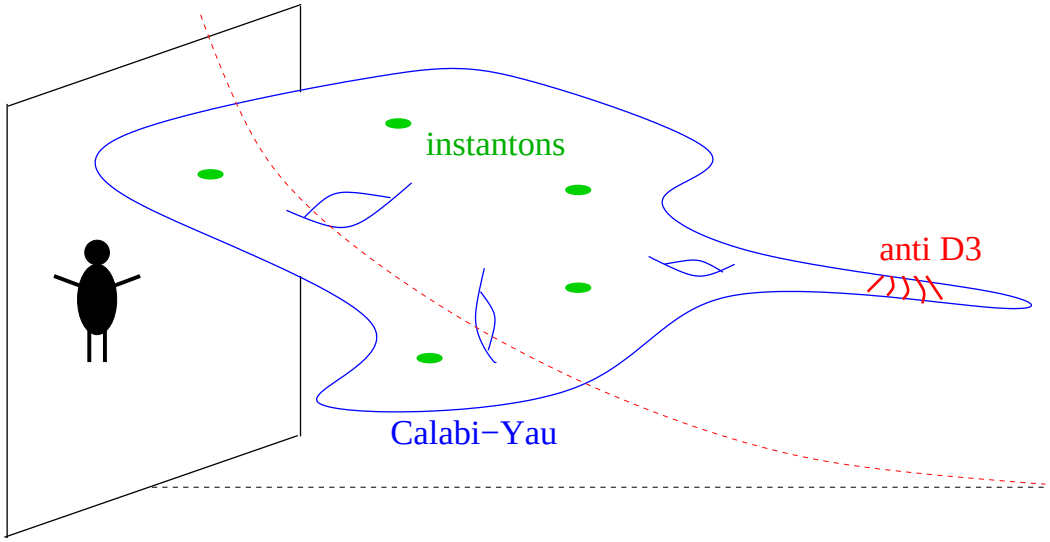
- what about backreaction?

Example: KKLT scenario for de Sitter solutions

[Kachru, Kallosh, Linde, Trivedi, 2003]



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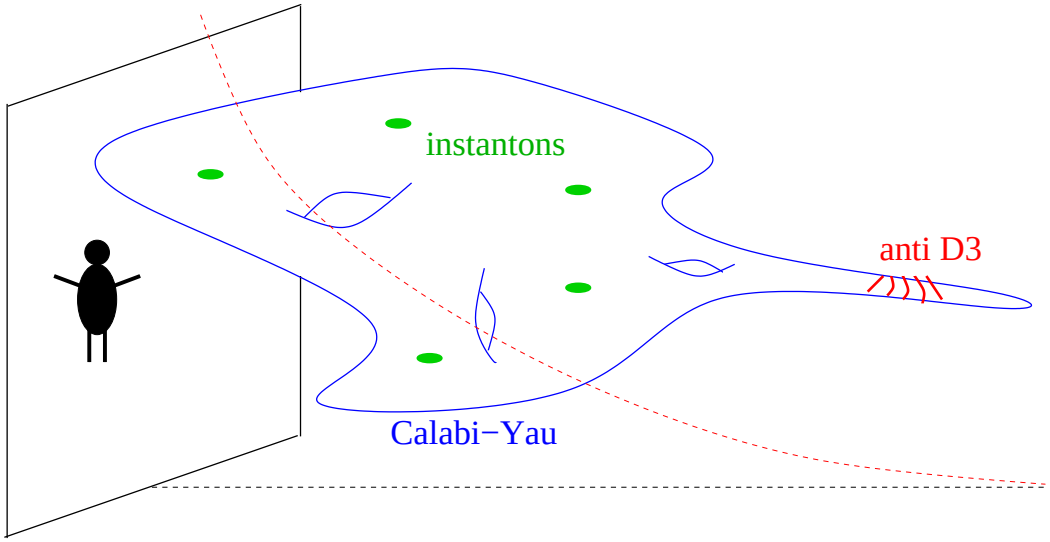

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However:

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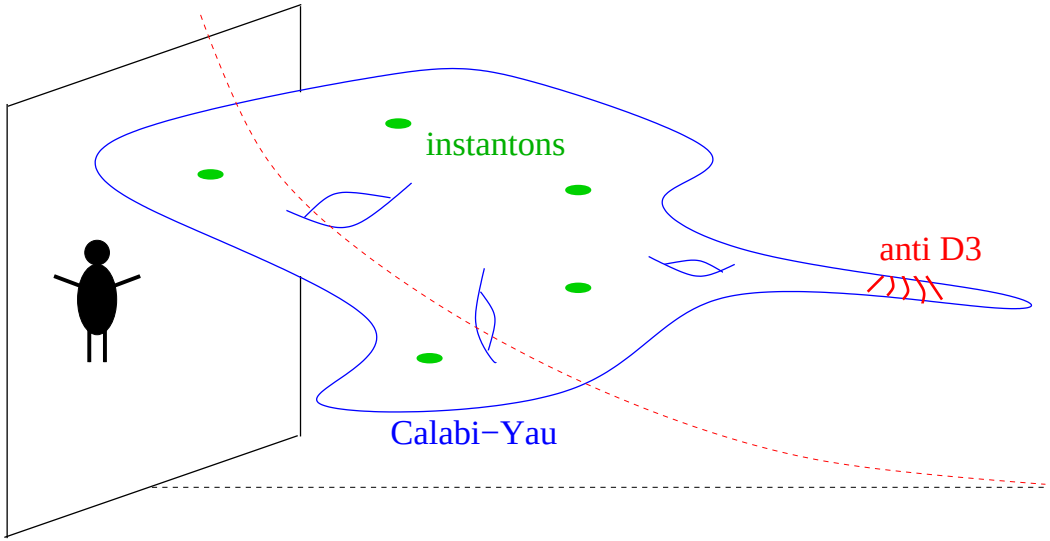

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However:

- No exact solution known $\longrightarrow$ delocalised (smeared) limit
- Broken supersymmetry $\longrightarrow$ instabilities?

$\longrightarrow$ Strong corrections expected
(Sometimes even cease to exist) [McOrist, Sethi, 2012]

Important to study backreaction!!

Study backreaction by:

1. find exact solutions to full SUGRA eqns of motion

2. construct Warped Effective Field Theory (WEFT)

   $\longrightarrow$ integrate over high-energy effects of warping

   $\longrightarrow$ construct low-energy effective action

BUT: what is low energy in presence of warping?

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Test case: Dynamical branes with extra fluxes

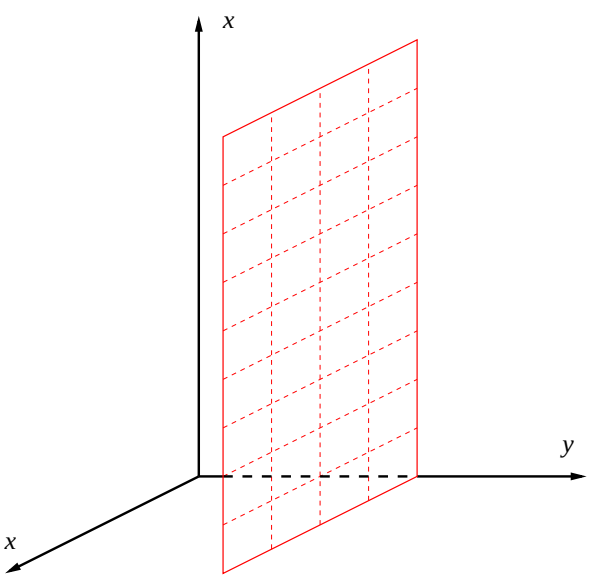
3. p-brane solutions

p-brane are fundamental objects in string theory
solitonic objects in supergravity

$$S = \int d^D x \sqrt{|g|} \Big[R \; + \; \partial_\mu \phi \partial^\mu \phi \; + \; e^{a\phi} \, F_{\mu_1 \ldots \mu_{p+2}} F^{\mu_1 \ldots \mu_{p+2}} \Big]$$

$$ds^2 = \mathcal{H}^\alpha(r) \eta_{mn} dx^m dx^n - \mathcal{H}^\beta(r) \Big[dr^2 + r^2 d\Omega^2 \Big], \qquad e^{-2\phi} = \mathcal{H}^\gamma(r),$$

$$F_{m_1 \ldots m_{p-1} i} = \varepsilon_{m_1 \ldots m_{p-1}} \partial_i \mathcal{H}^{-1}(r), \qquad \mathcal{H}(r) = 1 + \frac{M}{r^{D-p-3}}$$



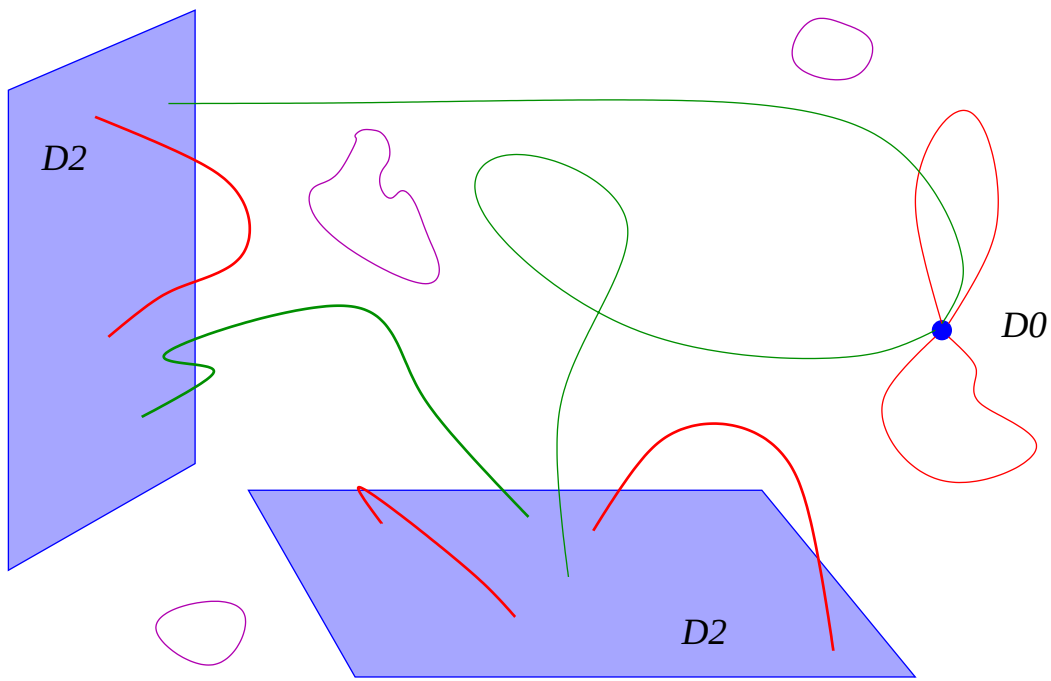
- Exact solution for α, β, γ in function of a, D, p
- Planar objects, extended in p spatial directions
- Electrically/magnetically charged under F_{p+2}
- Preserve some fraction of supersymmetry
- Generalization of electron in Maxwell theory, or black hole in General Relativity

Fundamental string (F1): cosmic string, fundamental object with $M \sim g$

[Dabholkar, Gibbons, Harvey, Ruiz Ruiz, 1990]

NS5-brane (NS5): solitonic object (cfr Dirac monopole)
with magnetic charge and $M \sim \frac{1}{g^2}$ [Callan, Harvey, Strominger, 1990]

D-branes (Dp): Dirichlet boundary conditions for open strings
with arbitrary p and $M \sim \frac{1}{g}$ [Polchinski, 1995]



Dualities: p-brane democracy: all branes are equally fundamental

[Townsend, 1995]

Rest of the talk: restrict to D6-brane in (m)IIA

$$S = \int d^{10}x\sqrt{|g|}\Big[R + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{6}e^{-\phi}H_{\mu\nu\rho}H^{\mu\nu\rho} - \tfrac{1}{4}e^{3\phi/2}\,F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}e^{5\phi/2}m^2\Big]$$

with

$F_{\mu\nu} = 2\partial_{[\mu}C_{\nu]} + mB_{\mu\nu}$ invariant under $\delta C_\mu = \partial_\mu\Lambda - m\Sigma_\mu$

$H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}$ invariant under $\delta B_{\mu\nu} = \partial_{[\mu}\Sigma_{\nu]}$

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Standard D6-brane (1/2 supersymmetric):

$$ds^2 = \mathcal{H}^{-\frac{1}{2}}(r)\,\eta_{ab}dx^a dx^b - \mathcal{H}^{\frac{7}{8}}(r)\Big[dr^2 + r^2 d\Omega_2^2\Big],$$

$$e^{-2\phi} = \mathcal{H}^{-\frac{3}{4}}(r), \qquad F_{\theta\varphi} = \partial_r\mathcal{H}^{-1}(r),$$

$$H_{\mu\nu\rho} = 0 = m, \qquad \bar{\nabla}^2\mathcal{H}(r) = 0$$

$\longrightarrow$ Cfr Dirac monopole in 10 dimensions

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   $\longrightarrow$ test with dynamical branes with extra fluxes

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4. Fractional dynamical branes

Dynamical p-branes:

[Kodama, Uzawa, 2005]

Dynamical branes = branes with extra WV dependence: $\mathcal{H} = \mathcal{H}(x, r)$

$$ds^2 = \mathcal{H}^{-\frac{1}{2}}(x,r)\,\eta_{ab}(x)dx^a dx^b - \mathcal{H}^{\frac{7}{8}}(x,r)\left[dr^2 + r^2 d\Omega_2^2\right],$$

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$$[g_{ij}] \;\Rightarrow\; \partial_i\partial^i\,\mathcal{H} = 0, \qquad [g_{ab}] \;\Rightarrow\; \partial_a\partial_b\,\mathcal{H} = 0, \qquad [g_{ai}] \;\Rightarrow\; \partial_i\partial_a\,\mathcal{H} = 0$$

4. Fractional dynamical branes

Dynamical p-branes:

[Kodama, Uzawa, 2005]

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Hence:

$$\begin{aligned}\mathcal{H}(x,r) &= \mathcal{H}_w(x) + \mathcal{H}_t(r) \\ &= c_a x^a + 1 + \frac{Q}{r}\end{aligned}$$

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Supersymmetric for specific value of c_a

What does linear WV dependence mean?

Interpretation: warped compactification

$$S = \int d^{10}x \sqrt{|g|} \Big[R + \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi \Big]$$

Smeared brane Ansatz ($Q = 0$):

$$ds_{10}^2 = e^{2\alpha\chi}\, \tilde{g}_{ab}(x) dx^a dx^b \; - \; e^{2\beta\chi} \Big[dr^2 \; + \; r^2 d\Omega_2^2 \Big]$$

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Calculations:

$$R = e^{-2\alpha\chi}\, \tilde{R} \; + \; e^{-2\alpha\chi} (\partial\chi)^2 \; + \; e^{-2\alpha\chi} \tilde{\nabla}^2 \chi$$

Reduced action:

$$S = \int d^8x \sqrt{|\tilde{g}|} \Big[\tilde{R} + \tfrac{1}{2} \partial_a \phi \partial^a \phi + \tfrac{1}{2} \partial_a \chi \partial^a \chi \Big]$$

$\longrightarrow$ Worldvolume dependence: 7-dimensional scalars

$\longrightarrow$ Smeared dynam D6: reduction to 7-dim domain wall with free scalars

$\longrightarrow$ Localised dynam D6: (trivial) backreaction of domain wall and scalars

Overview:

Standard D6

$\swarrow$

dynamical D6

$\mathcal{H} = \mathcal{H}(x, r)$

free scalar fields

Overview:

Standard D6

$\swarrow$ $\searrow$

dynamical D6

$\mathcal{H} = \mathcal{H}(x, r)$

free scalar fields

fractional D6

$dF_2 = mH_3 + Q$

Fractional p-branes:

[Klebanov, Strassler, 2000]

Fractional branes = branes with extra fluxes: $dF_2 = mH_3 + Q$

$$S = \int d^{10}x\sqrt{|g|}\Big[R + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{6}e^{-\phi}H_{\mu\nu\rho}H^{\mu\nu\rho} - \tfrac{1}{4}e^{3\phi/2}\,F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}e^{5\phi/2}m^2\Big]$$

Solution:

$$ds^2 = \mathcal{H}^{-\frac{1}{2}}(r)\,\eta_{ab}(x)dx^a dx^b - \mathcal{H}^{\frac{7}{8}}(r)\Big[dr^2 + r^2 d\Omega_2^2\Big],$$

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$$[g_{ij}] \;\Rightarrow\; \partial_i\partial^i\,\mathcal{H} = H_{r\theta\varphi}H^{r\theta\varphi}$$

Hence:

$$\mathcal{H}(r) \;=\; 1 + \frac{Q}{r} + \tfrac{1}{6}m^2r^2$$

BUT: No longer supersymetric!

Interpretation: warped compactification (again)

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Smeared brane Ansatz ($Q = 0$):

$$ds_{10}^2 = e^{2\alpha\chi}\,\tilde{g}_{ab}(x)dx^a dx^b \;-\; e^{2\beta\chi}\Big[dr^2 + r^2 d\Omega_2^2\Big]$$

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Calculations:

$$R = e^{-2\alpha\chi}\,\tilde{R} \;+\; e^{-2\alpha\chi}(\partial\chi)^2 \;+\; e^{-2\alpha\chi}\tilde{\nabla}^2\chi \qquad H_{\mu\nu\rho}H^{\mu\nu\rho} = e^{a\chi}\,h^2$$

Reduced action:

$$S = \int d^7x\sqrt{|\tilde{g}|}\Big[\tilde{R} + \tfrac{1}{2}\partial_a\phi\partial^a\phi + \tfrac{1}{2}\partial_a\chi\partial^a\chi - e^{a\chi+b\phi}h^2 + e^{c\phi}m^2\Big]$$

$\longrightarrow$ Non-trivial scalar potential: gauged SUGRA from flux compactification

$\longrightarrow$ Smeared fractional D6: reduction to 7-dim Minkowski space (no scalars)

$\longrightarrow$ Localised fractional D6: backreaction of orientifold flux compactification

Overview and strategy:

Standard D6

↙ ↘

dynamical D6
$\mathcal{H} = \mathcal{H}(x, r)$
free scalar fields

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fractional D6
$dF_2 = mH_3 + Q$
non-trivial potential

$\searrow$ $\swarrow$

fractional, dynamical D6
$\mathcal{H} = \mathcal{H}(x, r)$ & $dF_2 = mH_3 + Q$
scalars in non-trivial potential!

$\longrightarrow$ scalars probe potential!

Fractional dynamical p-branes:

[Blåbäck, B.J., Vercnocke, Van Riet, 2012]

Fluxes + worldvolume dependence:

$$S = \int d^D x \sqrt{|g|} \Big[R + \tfrac{1}{2}\partial_\mu \phi \partial^\mu \phi + \tfrac{1}{6} e^{-\phi} H_{\mu\nu\rho} H^{\mu\nu\rho} - \tfrac{1}{4} e^{3\phi/2}\, F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} e^{5\phi/2} m^2 \Big]$$

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Hence:

$$\begin{aligned} \mathcal{H}(x,r) &= \mathcal{H}_w(x) + \mathcal{H}_t(r) \\ &= mz + 1 + \frac{Q}{r} + \tfrac{1}{6} m^2 r^2 \end{aligned}$$

$\longrightarrow$ 1/4 supersymmetric

$\longrightarrow$ Still sum of linear WV part and transversal dependence

Interpretation: warped compactification

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We expect a supersymmetric domain wall solution with running scalars...
Let's look for it...

Most general susy domain wall of $SO(2)$ gauged D=7 maximal SUGRA

$$S = \int d^8x \sqrt{|\tilde{g}|} \Big[\tilde{R} + \tfrac{1}{2}\partial_a\phi\partial^a\phi + \tfrac{1}{2}\partial_a\chi\partial^a\chi - e^{a\chi+b\phi}h^2 + e^{c\phi}m^2 \Big]$$

is given by

[Bergshoeff, Nielsen, Roest, 2004]

$$ds_7^2 = (f_1\, f_2)^{\frac{1}{10}}\eta_{ij}dx^i dx^j \;-\; (f_1\, f_2)^{-\frac{2}{5}}dz^2$$

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Change of variables: $SO(2)$ rotation $(\phi, \chi) \longrightarrow (x, u)$

$$V \;=\; h^2 e^{a\chi+b\phi} \;-\; m^2 e^{c\phi} \;=\; e^{\gamma x}\Big[h\, e^{-u} \;-\; m\, e^{u}\Big]^2$$

Special case: in the minimum of the potential

$$V = 0 \iff e^{2u} = \frac{h}{m} \iff f_1 = \frac{h}{m} f_2$$

$\longrightarrow$ precisely our case of smeared dynamical fractional D6!

Our case:

$$ds^2 = \mathcal{H}^{-\frac{1}{2}}(x,r)\,\eta_{ab}(x)dx^a dx^b - \mathcal{H}^{\frac{7}{8}}(x,r)\Big[dr^2 + r^2 d\Omega_2^2\Big],$$

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$$\mathcal{H}(x,r) \;=\; \mathcal{H}_w(x) \;+\; \mathcal{H}_t(r) \;=\; mz \;+\; 1 + \frac{Q}{r} \;+\; \tfrac{1}{6}m^2r^2$$

$\longrightarrow$ Extra fluxes: Non-trivial potential of gauged supergravity

Worldvolume dependence induce running scalars

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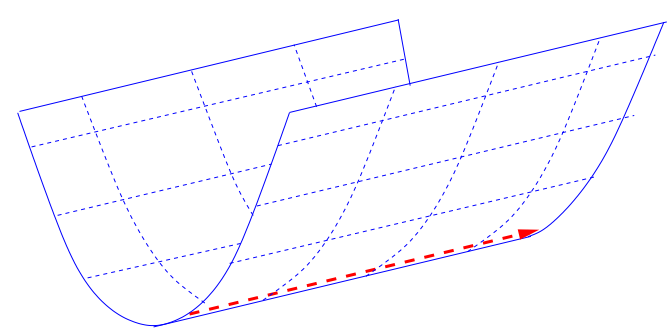
Worldvolume dependence induce running scalars

$\longrightarrow$ Smeared dynam D6: reduction to 7-dim domain wall with scalars

$\longrightarrow$ Localised dynam D6: backreaction of domain wall and scalars

$\longrightarrow$ scalars run in minimum of potential:
free on shell! $\Rightarrow$ linear dependence!



5. General backreacted domain walls

Dynamical fractional brane is special case $f_1 = \frac{h}{m} f_2$ of general domain wall

$$ds_7^2 = (f_1\, f_2)^{\frac{1}{10}} \eta_{ij} dx^i dx^j \; - \; (f_1\, f_2)^{-\frac{2}{5}} dz^2$$

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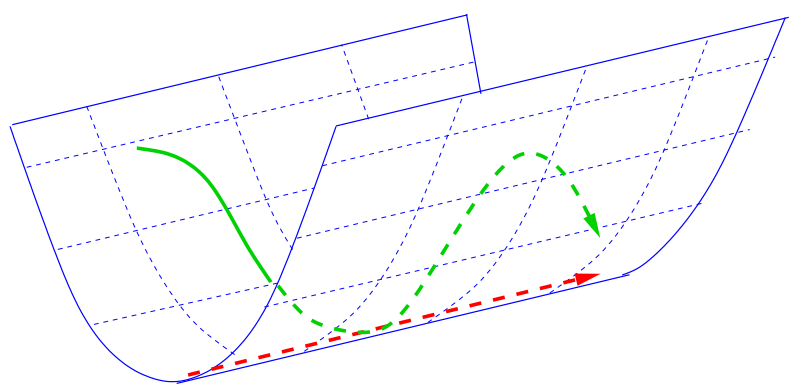
with a scalar running in the minimum of the potential.

Then $f_1 \neq \frac{h}{m} f_2$ represents supersymmetric domain walls with non-trivial running scalar



$\longrightarrow$ Localised solution: Backreaction of running scalar

$\longrightarrow$ Supersymmetric restricts form of Ansatz

Most general supersymmetric Ansatz of this form [Imamura, 2001]

$$ds^2 = S^{-1/2}\eta_{ij}dx^i dx^j + KS^{-1/2}dz^2 + KS^{1/2}\left[dr^2 + r^2 d\Omega_2^2\right],$$

$$e^{\phi} = K^{1/2}S^{-3/4}, \qquad F_{\theta\varphi} = -\partial_r S, \qquad H_{r\theta\varphi} = \partial_{\tilde{z}}(KS), \qquad H_{z\theta\varphi} = \partial_r K,$$

with

$$\nabla^2 S + \tfrac{1}{2}\partial_z^2 S^2 = -Q_6\delta, \qquad mg_s K = \partial_z S$$

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with

$$\nabla^2 S + \tfrac{1}{2}\partial_z^2 S^2 = -Q_6\delta, \qquad mg_s K = \partial_z S$$

$\longrightarrow$ Determine K and S such that the solution reduces to domain wall solution in smeared case:

Laurent series: $S(r,\tilde{z}) = \sum_{n=-1}^{\infty} a_n(z) r^n$

Imamura eqns: $n(n+1)a_n = -\frac{1}{2}\partial_z^2\left(\sum_{k=0}^{n} a_{k-1}a_{n-k-1}\right)$

z-independent charge: $a_{-1} = Q$

in smeared case $H_{r\theta\varphi} = h$: $\frac{1}{2}\partial_z^2 a_0^2 = mh$

Solution no too illuminating:

$S(r,\tilde{z}) = \sum_{n=-1}^{\infty} a_n(z) r^n$
$m g_s K = \partial_z S$ with

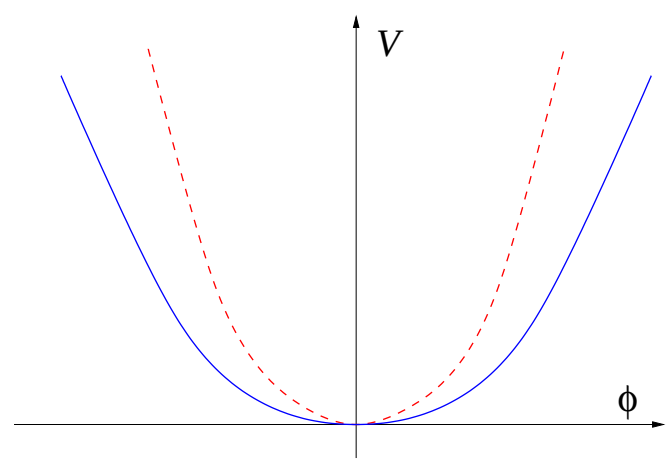
$$a_1 = -\beta \frac{g_s m h Q_6}{2a_0^3}$$

$$a_2 = -\frac{1}{6} g_s m h + \beta (g_s^2 m h Q_6)^2 \left(\frac{1}{a_0^5} - \frac{5\beta}{4a_0^7} \right)$$

$$a_3 = \beta (g_s m h)^2 (g_s Q_6) \left(\frac{1}{4a_0^4} - \frac{\beta}{3a_0^6} \right) + \beta (g_s^2 m h Q_6)^3 \left(-\frac{5}{2a_0^7} + \frac{35\beta}{4a_0^9} - \frac{105\beta^2}{16a_0^{11}} \right)$$

...

Warped effective potential and gauged SUGRA potential coincide in minimum



6. Summary and outlook

• Flux compactifications can lead to interesting results, if done correctly, taking backreaction in account!

⋆ Adding fluxes leads to lower-dim theories with non-trivial potentials

⋆ Worldvolume dependences leads to lower-dim dynamical scalars

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  - ⋆ Worldvolume dependences leads to lower-dim dynamical scalars

- Fractional dynamical branes describe on-shell free scalars

$$\mathcal{H}(x,r) \;=\; \mathcal{H}_w(x) \;+\; \mathcal{H}_t(r) \;=\; mx \;+\; 1 + \frac{Q}{r} \;+\; \tfrac{1}{6}m^2r^2$$

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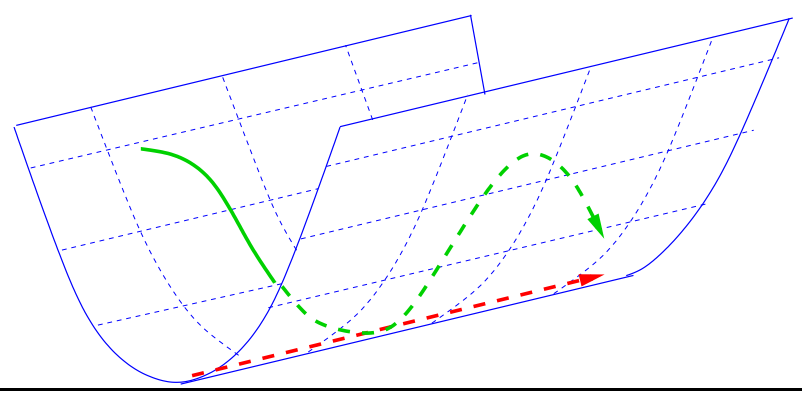
- General supersymmetric brane solutions with non-trivial running scalars

$$\mathcal{H}(x,r) \;\neq\; \mathcal{H}_w(x) \;+\; \mathcal{H}_t(r)$$



- What about non-supersymmetric case?

  ⋆ Fractional dynamical D6 found in 1998: [B.J., Meessen, Ortín, 1998]

$$\mathcal{H}(x,r) \;=\; c_a x^a \;+\; 1 + \frac{Q}{r} \;+\; \tfrac{1}{6} m^2 r^2$$

solution for general $\mathcal{H}_w = c_a x^a$, but supersymmetric for $\mathcal{H}_w = mx$

$\longrightarrow$ what do other solutions represent?

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$\longrightarrow$ what do other solutions represent?

⋆ Supersymmetric solutions come from superpotencial W:

$$\begin{aligned} V &= e^{2\gamma x}\Big[h\, e^{-u} \;-\; m\, e^{u}\Big]^2 \\ &= \tfrac{1}{2}(\partial_x W)^2 \;+\; \tfrac{1}{2}(\partial_u W)^2 \;-\; \tfrac{3}{10} W^2 \end{aligned}$$

with

$$W = e^{\gamma x}\Big[h\, e^{-u} \;+\; m\, e^{u} + C\Big]$$

$C = 0$: real superpotential, real supersymmetry transf.

$C \neq 0$: fake superpotential, only formal transform.

$\longrightarrow$ Strong enough to restrict solutions?

Thank you!