Black Hole Thermodynamics

Bert Janssen
Dpto. de Física Teórica y del Cosmos
Centro Andaluz de Física de Partículas Elementales

Outlook

1. General Relativity: a quick review

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2. The Schwarzschild black hole
3. The Kerr black hole

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4. Penrose Process

5. The area theorem

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6. The laws of black hole mechanics
7. Quantum black holes:
   $\longrightarrow$ Counting of microstates
   $\longrightarrow$ Information paradox

Disclaimer:

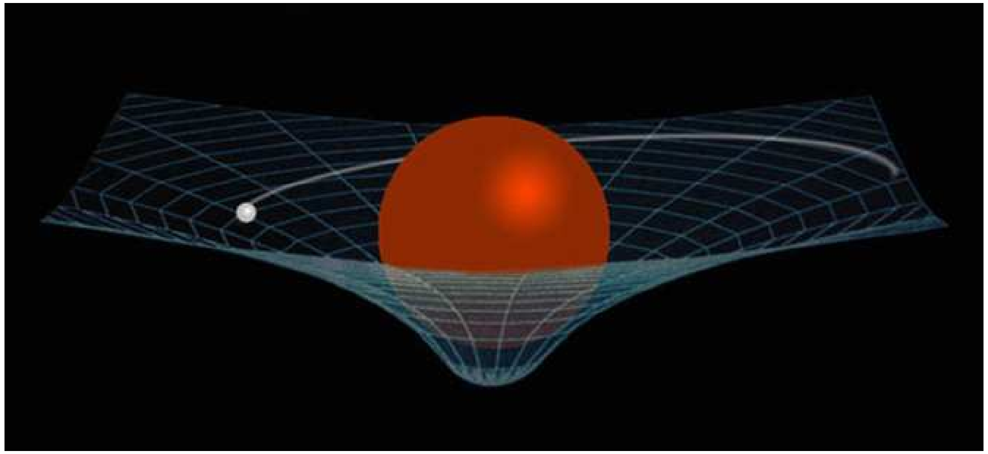
Main goal of this talk is to expose open issues ,
not to propose concrete solutions...

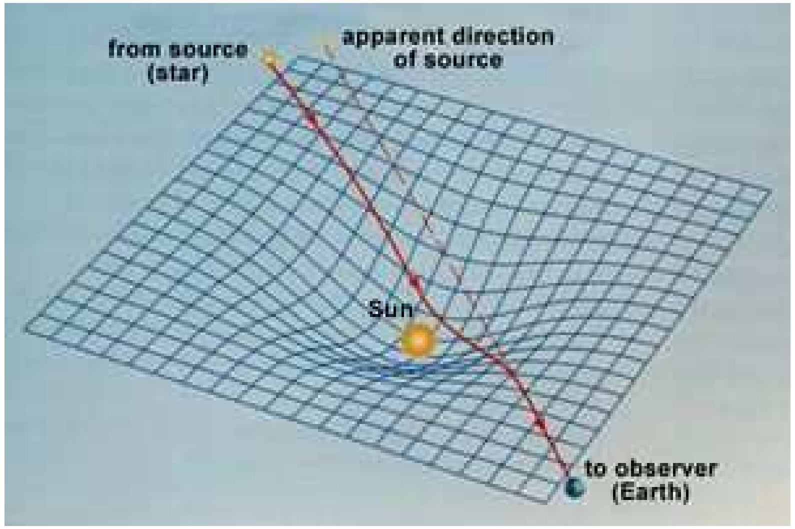
0. Selected bibliography

- T. Jacobson, *Introductory Lectures on Black Hole Thermodynamics*
- S. Mathur, *The information paradox: a pedagogical introduction*, `arXiv:0909.1038 [hep-th]`
- Misner, Thorne & Wheeler, *Gravitation*, 1970
- E. Poisson, *A Relativist's Toolkit*, Cambridge University Press, 2004
- P. Townsend, *Black Holes*, `arXiv:gr-qc/9707012`
- R. Wald, *Quantum field theory in curved spacetime and black hole theormodynamics*, Bangalore Pres, 1994
- R. Wald, *The Thermodynamics of Black Holes*, `arXiv:gr-qc/9912119`
- Wikipedia (english): Black Hole thermodynamics, Information paradox, Holographic Principle, ...

1. General Relativity: a quick review

Gravity = manifestation of curved spacetime





John A. Wheeler: *Matter says space how to curve*
Space says matter how to move

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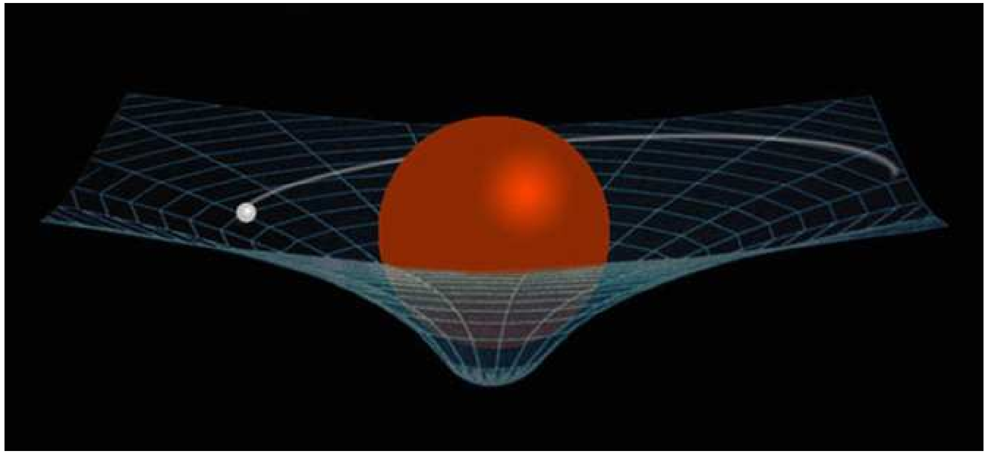

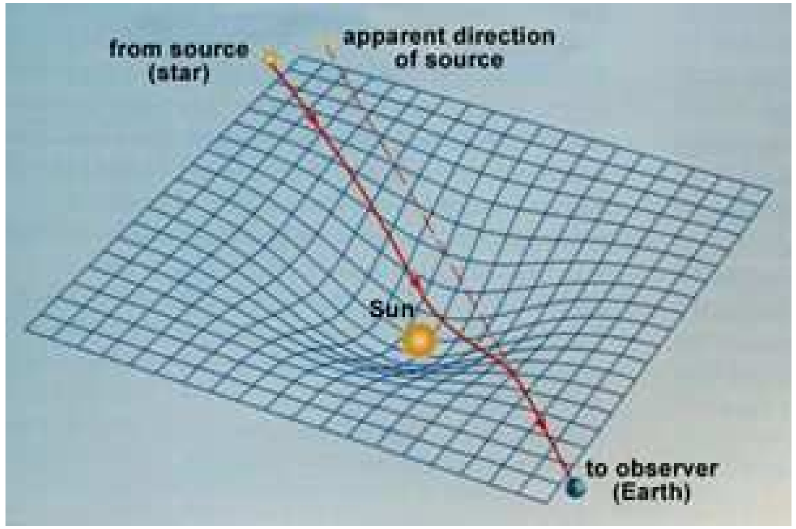

John A. Wheeler: *Matter says space how to curve*
Space says matter how to move

- Spacetime = 4-dim Lorentzian manifold, equiped with metric $g_{\mu\nu}$ and Levi-Civita connection $\Gamma^{\rho}_{\mu\nu}$.

$$g_{\mu\nu} \quad \Longrightarrow \quad \Gamma^{\rho}_{\mu\nu} \quad \Longrightarrow \quad R_{\mu\nu\rho}{}^{\lambda}$$

- Equivalence Principle: Weight can be locally gauged away
  Inhomogeneities of gravitational field are shown in tidal effects

$$\text{weight} \ \sim \ \Gamma_{\mu\nu}^{\rho}, \qquad \qquad \text{tidal forces} \ \sim \ R_{\mu\nu\rho}{}^{\lambda}$$

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- Einstein equations: relation between curvature and matter content

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R = -\kappa \Big[F_{\mu\rho} F_{\nu}{}^{\rho} - \tfrac{1}{4} g_{\mu\nu} F_{\rho\lambda} F^{\rho\lambda} \Big] - \frac{\kappa m}{4\pi} u^{\mu} u^{\nu} \delta(x - x(\tau))$$
$$\nabla_{\mu} F^{\mu\nu} = q \dot{x}^{\nu}\, \delta(x - x(\tau))$$
$$m \Big(\ddot{x}^{\rho} + \Gamma_{\mu\nu}^{\rho} \dot{x}^{\mu} \dot{x}^{\nu} \Big) = q\, \dot{x}_{\mu} F^{\mu\rho}$$

$\longrightarrow$ system of $10 + N$ non-linear coupled 2nd order partial diff eqns
for $g_{\mu\nu}(x)$ and $A_{\mu}(x)$ and $x^{\mu}(\tau)$

$\longrightarrow$ in general extremely difficult to solve!

2. The Schwarzschild black hole

Metric of static spherically symmetric vacuum solution [Schwarzschild, 1916]

$$ds^2 = \Big(1 - \frac{2M}{r}\Big)dt^2 - \Big(1 - \frac{2M}{r}\Big)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- (External region of) spherically symmetric object with mass $m = M/G$
  In GR, mass is only asymptotically defined: $g_{tt} \approx 1 + Gmr^{-1} + ...$
- Singular for $r = 0$ and $r = 2M$:

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  Curvature invariante $R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} = 48\,G^2\,m^2\,r^{-6}$
  $\longrightarrow$ $r = 0$ is a physical singularity
  is point of infinite tidal forces

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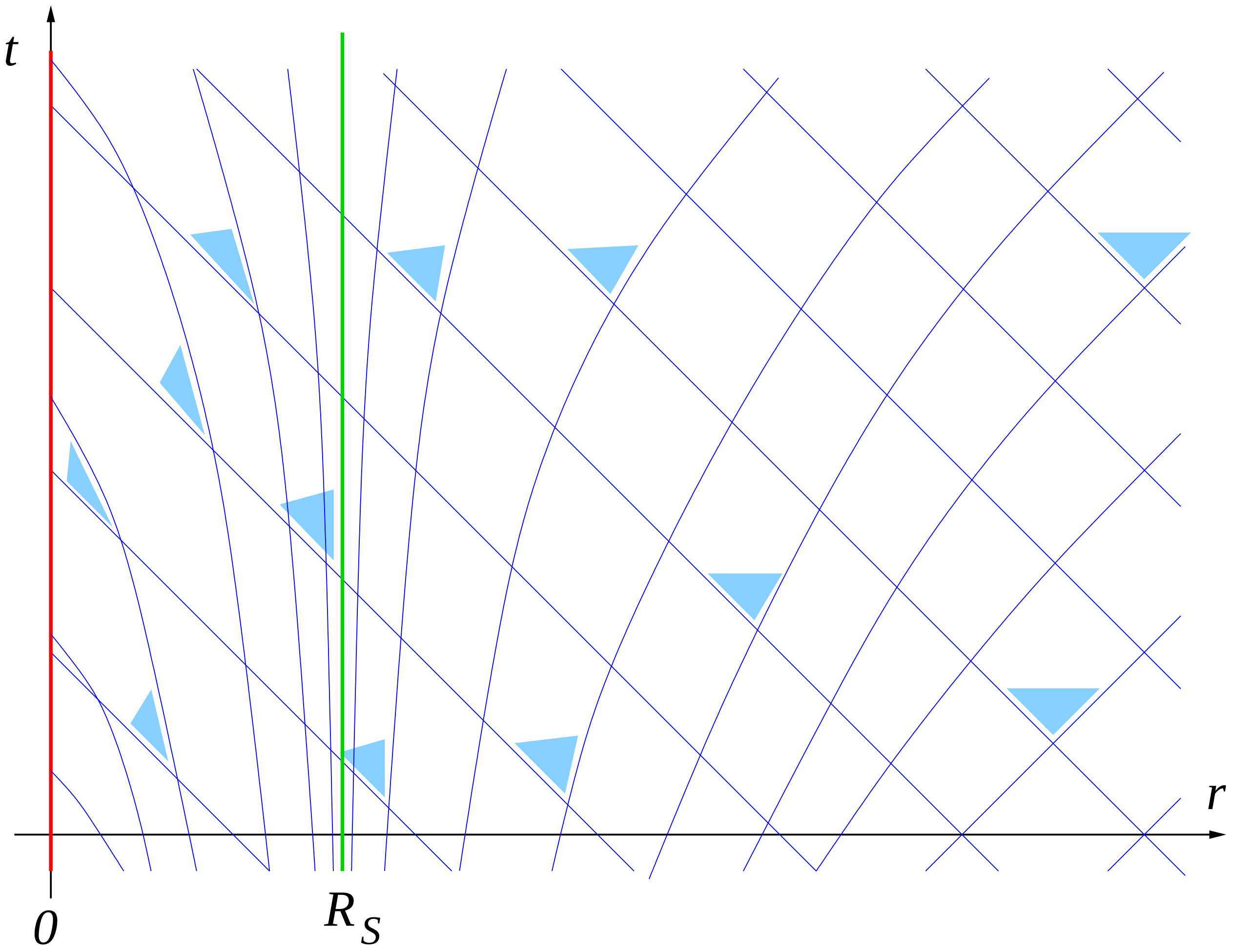
  $\longrightarrow$ $r = 0$ is a physical singularity
  is point of infinite tidal forces
  $\longrightarrow$ $r = 2M$ is a coordinate singularity
  $\longrightarrow$ Schwarzschild radius

- Eddington-Finkelstein coordinates: $\tilde{t} = t + 2M\log(r - 2M)$

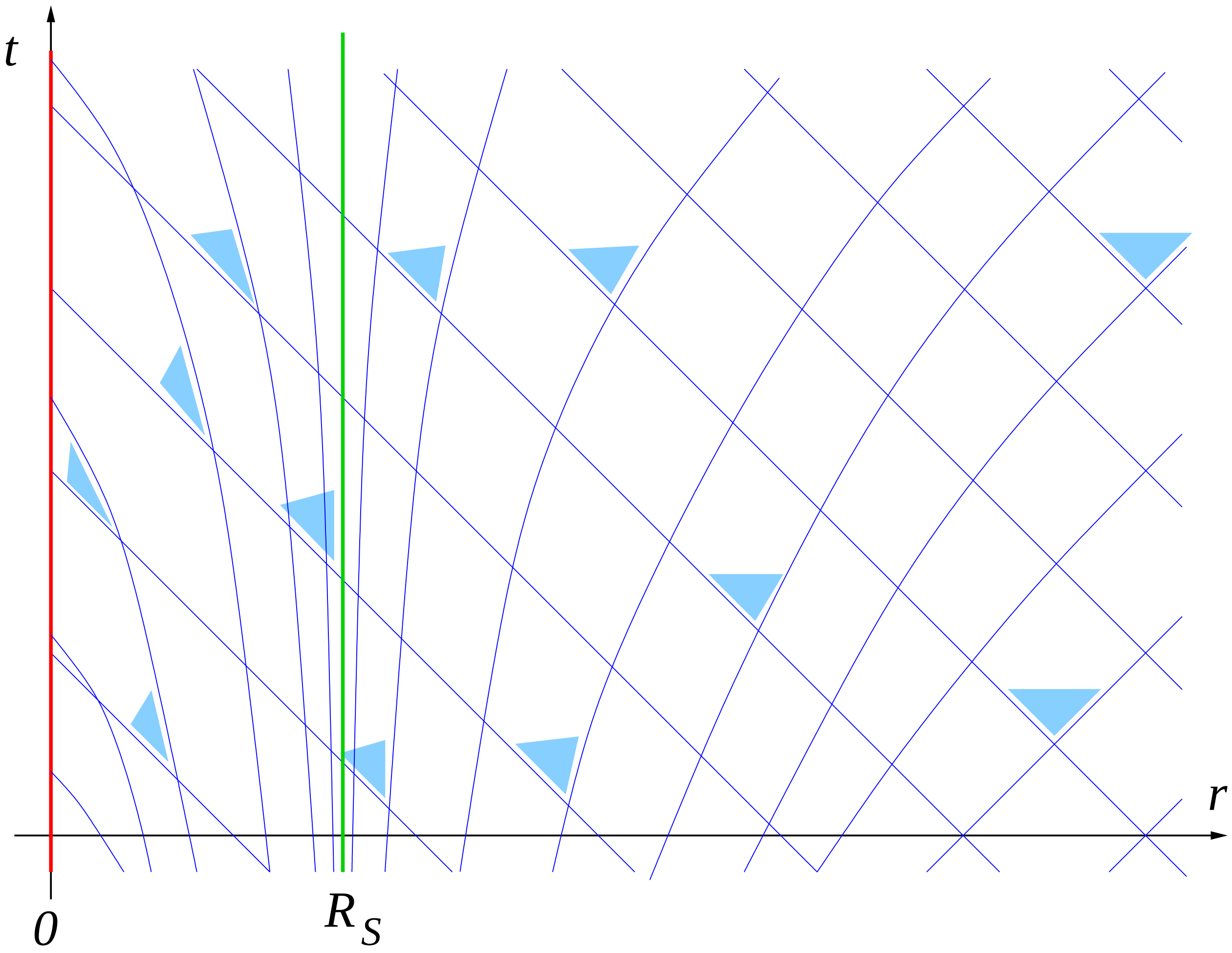
$$ds^2 = \Big(1 - \frac{2M}{r}\Big)d\tilde{t}^2 - \frac{4M}{r}d\tilde{t}dr - \Big(1 + \frac{2M}{r}\Big)dr^2 - r^2 d\Omega_2^2$$



$\longrightarrow$ Lightcones incline towards singularity

$\longrightarrow$ $r = 2M$: surface of infinite redshift

$\longrightarrow$ $r = 2M$ point of no return: light and matter end inevitably in singularity



$\longrightarrow$ Lightcones incline towards singularity

$\longrightarrow$ $r = 2M$: surface of infinite redshift

$\longrightarrow$ $r = 2M$ point of no return: light and matter end inevitably in singularity

$\longrightarrow$ No causal influences can travel from $r < 2M$ to $r > 2M$ [Finkelstein, 1958]

$\longrightarrow$ $r = 2M$ is an event horizon: one-dimensional causal membrane

3. The Kerr solution

Metric of stationary axially symmetric vacuum solution [Kerr, 1963]

$$
\begin{aligned}
ds^2 &= \frac{r^2 - 2Mr + a^2\cos^2\theta}{r^2 + a^2\cos^2\theta}\,dt^2 + \frac{4Mar\sin^2\theta}{r^2 + a^2\cos^2\theta}\,dtd\varphi - \frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2}\,dr^2 \\
&\quad - (r^2 + a^2\cos^2\theta)\,d\theta^2 - \Big[r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{r^2 + a^2\cos^2\theta}\Big]\sin^2\theta\,d\varphi^2
\end{aligned}
$$

- Rotating, axially symmetric object with mass $m = M/G$ and angular momentum $J = a/M$
- Unique solution with these characteristics!

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$$- (r^2 + a^2\cos^2\theta)\, d\theta^2 - \Big[r^2 + a^2 + \frac{2Ma^2 r\sin^2\theta}{r^2 + a^2\cos^2\theta}\Big] \sin^2\theta\, d\varphi^2$$

- Rotating, axially symmetric object with mass $m = M/G$ and angular momentum $J = a/M$
- Unique solution with these characteristics!

Kerr-Newmann (idem with charge Q) is unique solution of stationary black hole! $\longrightarrow$ Uniqueness theorems [Israel; Hawking; Carter; 1965-1975]

- stationary $\Longrightarrow$ axially symmetric $\Longrightarrow$ Kerr-Newmann
- static $\Longrightarrow$ spherically symmetric $\Longrightarrow$ Reissner-Nordström

Black hole have no hair: completely determined by M, Q and J!

$$
\begin{aligned}
ds^2 \;=\; & \frac{r^2 - 2Mr + a^2\cos^2\theta}{r^2 + a^2\cos^2\theta}\, dt^2 \;+\; \frac{4Mar\sin^2\theta}{r^2 + a^2\cos^2\theta}\, dt d\varphi \;-\; \frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2}\, dr^2 \\
& -\; (r^2 + a^2\cos^2\theta)\, d\theta^2 \;-\; \Big[r^2 + a^2 + \frac{2Ma^2 r\sin^2\theta}{r^2 + a^2\cos^2\theta} \Big] \sin^2\theta\, d\varphi^2
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- Physical singularity at $r = a$ and $\theta = 0$ $\longrightarrow$ ring singularity!

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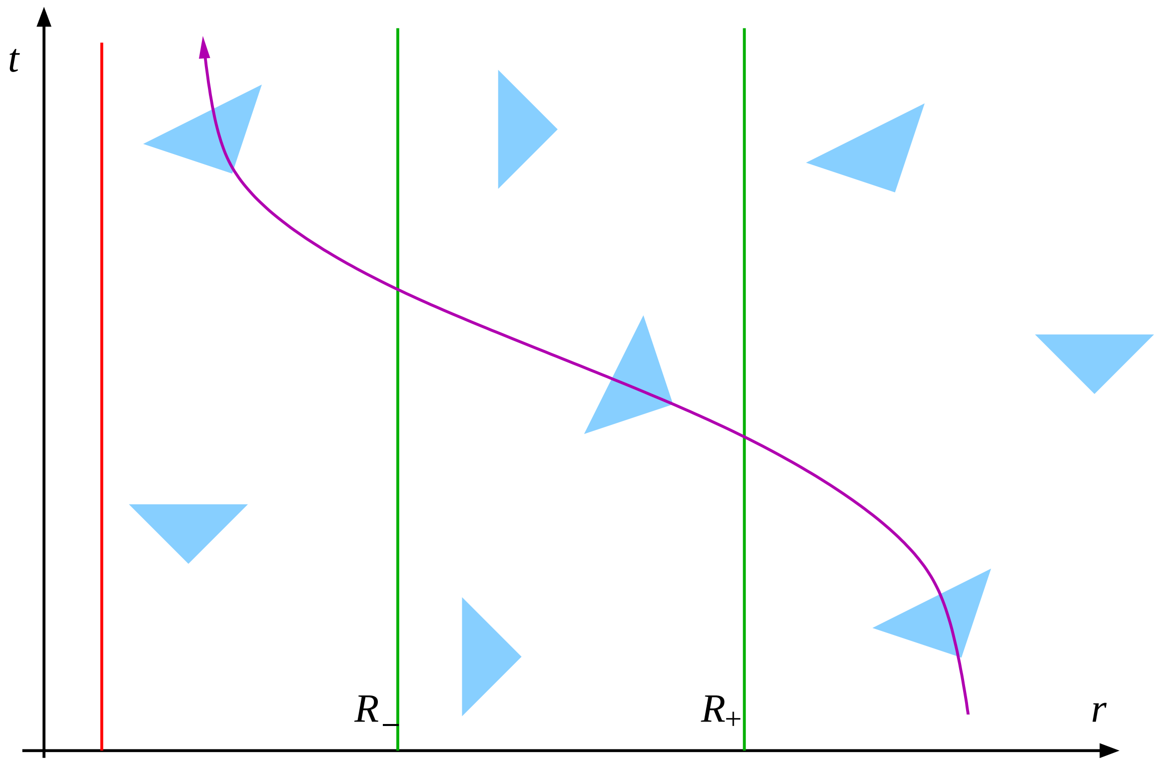
$$- (r^2 + a^2\cos^2\theta)\, d\theta^2 - \Big[r^2 + a^2 + \frac{2Ma^2 r\sin^2\theta}{r^2 + a^2\cos^2\theta}\Big] \sin^2\theta\, d\varphi^2$$

- Physical singularity at $r = a$ and $\theta = 0$ $\longrightarrow$ ring singularity!
- $g^{rr} = 0$: Inner and outer horizon at $r = M \pm \sqrt{M^2 - a^2} \equiv R_\pm$

  $\longrightarrow$ limit of static region



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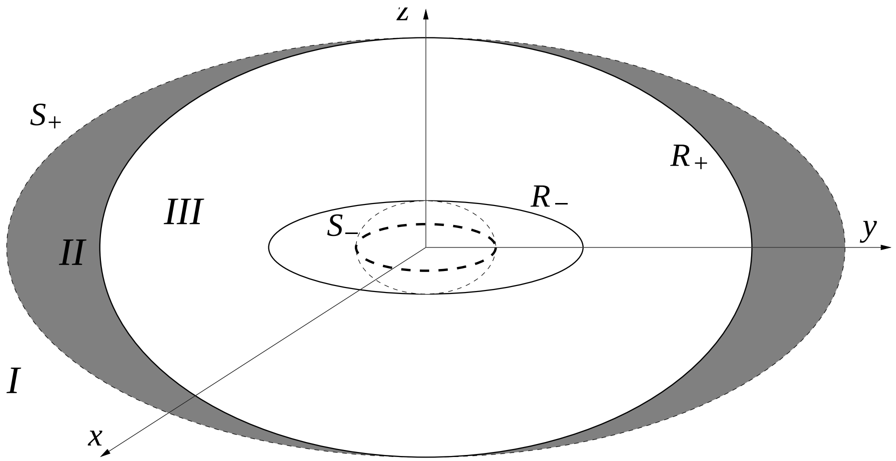
- Physical singularity at $r = a$ and $\theta = 0 \longrightarrow$ ring singularity!
- $g^{rr} = 0$: Inner and outer horizon at $r = M \pm \sqrt{M^2 - a^2} \equiv R_\pm$
  $\longrightarrow$ limit of static region
- $g_{tt} = 0$: Surfaces of infinite redshift at $r = M \pm \sqrt{M^2 - a^2\cos^2\theta} \equiv S_\pm$
  $\longrightarrow$ limit of stationary region
  $\longrightarrow$ Effects of frame dragging



Frame dragging:

[Lense & Thirring, 1918]

General axially symmetric stationary metric:

$$ds^2 = g_{tt}\, dt^2 + 2g_{t\varphi}\, dt d\varphi + g_{rr}\, dr^2 + g_{\theta\theta}\, d\theta^2 + g_{\varphi\varphi}\, d\varphi^2$$

Stationary observer: rotation with constant velocity in equatorial plane

Frame dragging:

[Lense & Thirring, 1918]

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$$ds^2 \;=\; g_{tt}\,dt^2 \;+\; 2g_{t\varphi}\,dtd\varphi \;+\; g_{rr}\,dr^2 \;+\; g_{\theta\theta}\,d\theta^2 \;+\; g_{\varphi\varphi}\,d\varphi^2$$

Stationary observer: rotation with constant velocity in equatorial plane

$\longrightarrow$ Angular velocity: $\Omega \;=\; \frac{d\varphi}{dt} \;=\; \frac{\dot{\varphi}}{\dot{t}}$

$\longrightarrow$ Angular momentum: $L \;=\; -g_{\varphi\mu}p^\mu \;=\; -m_0\,(g_{t\varphi}\,\dot{t} \;+\; g_{\varphi\varphi}\,\dot{\varphi})$

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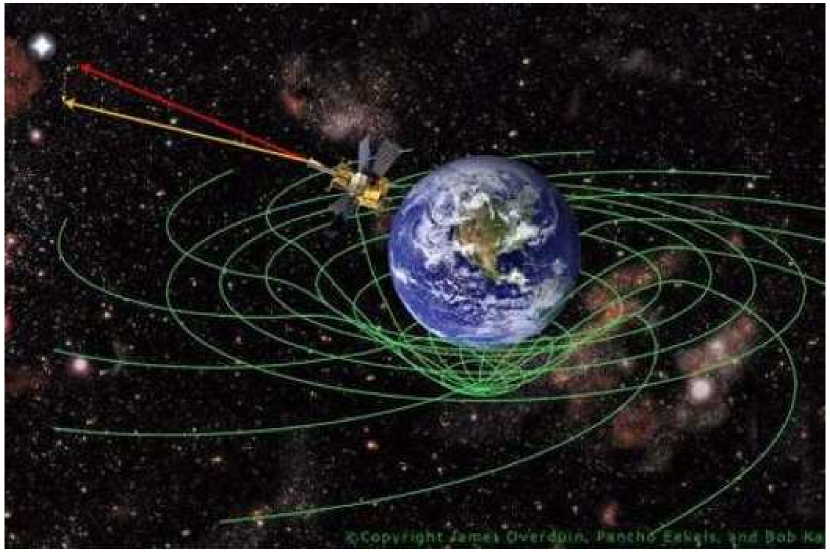
Zero angular momentum observer: dragged along with rotation

$$L \;=\; 0 \qquad \Longrightarrow \qquad \Omega_0 \;=\; -\frac{g_{t\varphi}}{g_{\varphi\varphi}} \;\neq\; 0$$



Lense-Thirring effect for Earth:

Prediccion: $-39,2$ mili-arcsec/year

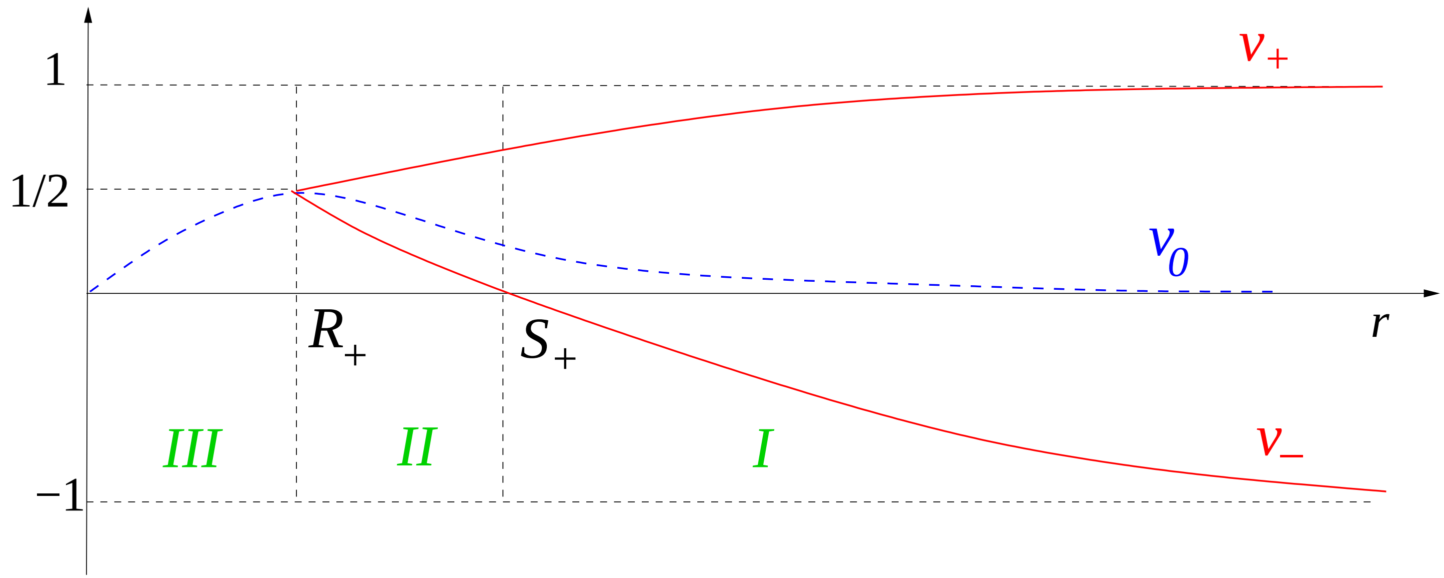
GPB: $-37,2 \,\pm\, 7,2$ mili-arcsec/year

[Gravity Probe B, 2011]

Ergoregion: Frame dragging limits velocities of stationary observers

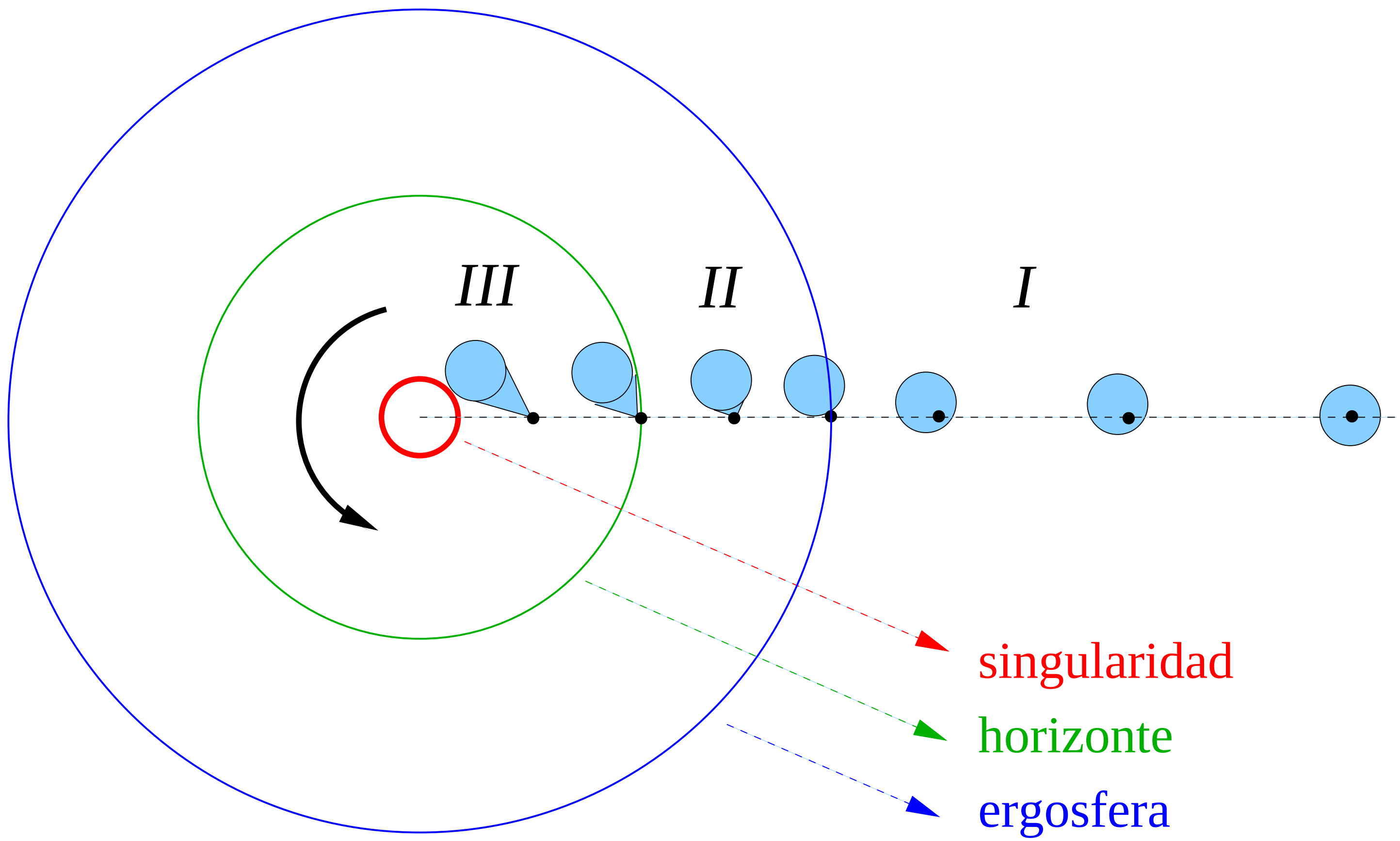
$$\Omega_{\pm} = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} \pm \sqrt{\left(\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right)^2 - \frac{g_{tt}}{g_{\varphi\varphi}}}$$

$$= \frac{2Ma}{r^3+a^2r+2Ma^2} \pm \sqrt{\frac{4M^2a^2}{(r^3+a^2r+2Ma^2)^2} + \frac{r-2M}{r^3+a^2r+2Ma^2}} \qquad \text{(Kerr)}$$



- Region I ($g_{tt} > 0$): both co-rotation and counter-rotation possible
- Region II ($g_{tt} < 0$, $g^{rr} > 0$): only co-rotation possible
- Region III ($g^{rr} < 0$): no stable rotation

Equatorial plane:



Ergoregion is not trapped surface!

- Region I: static observers; escape to infinity
- Region II: stationary non-static observers; escape to infinity
- Region III: non-stationary, trapped observers

4. The Penrose process

Energy at infinity (seen by asymptotic observer)

$$E = t_\mu p^\mu = g_{t\mu} p^\mu = g_{tt} p^t + g_{t\varphi} p^\varphi$$

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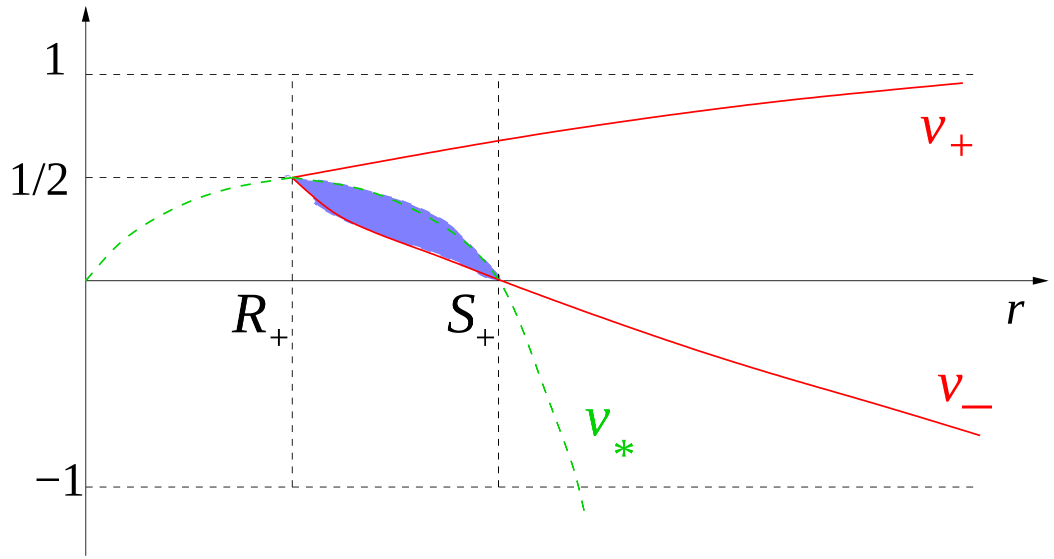
In ergoregion: $g_{tt} < 0$:

$$E < 0 \qquad \Longleftrightarrow \qquad p^\varphi < -\frac{g_{tt}}{g_{t\varphi}} p^t$$

$\longrightarrow$ opposing frame dragging as much as possible



NB: $E < 0$ only for asymptotic observer. Local observer sees $E > 0$!

Penrose process: extracting energy from black hole! [Penrose, 1969]

Energy and angular momentum conservation: $A \longrightarrow B + C$

$$E_A = E_B + E_C, \qquad\qquad L_A = L_B + L_B$$

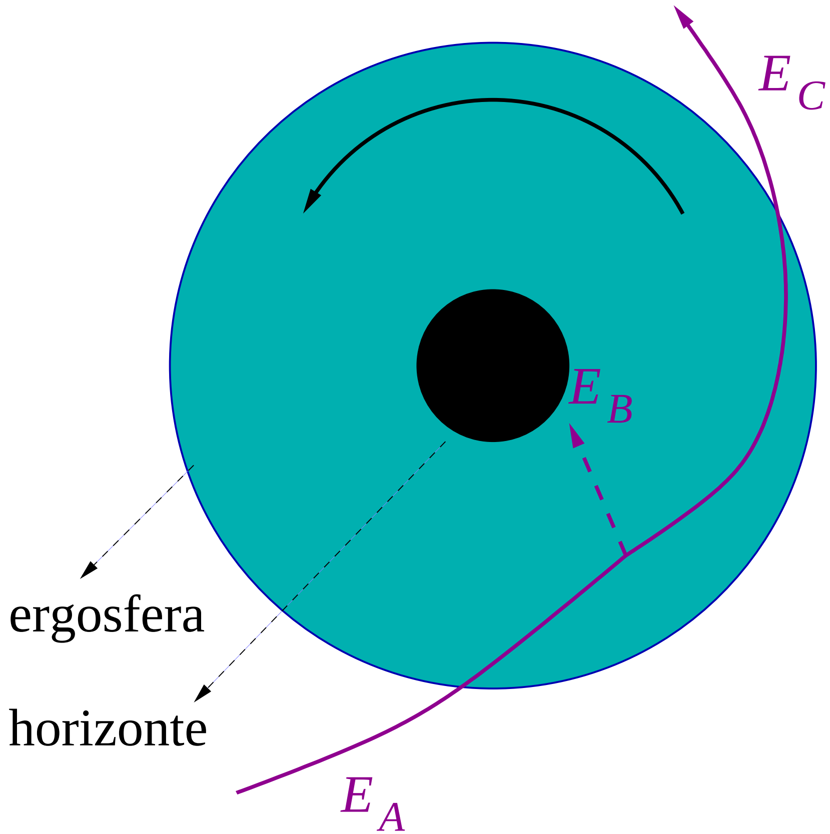
Penrose process: extracting energy from black hole! [Penrose, 1969]

Energy and angular momentum conservation: $A \longrightarrow B + C$

$$E_A = E_B + E_C, \qquad L_A = L_B + L_B$$

Launch particle such that $E_B < 0$:

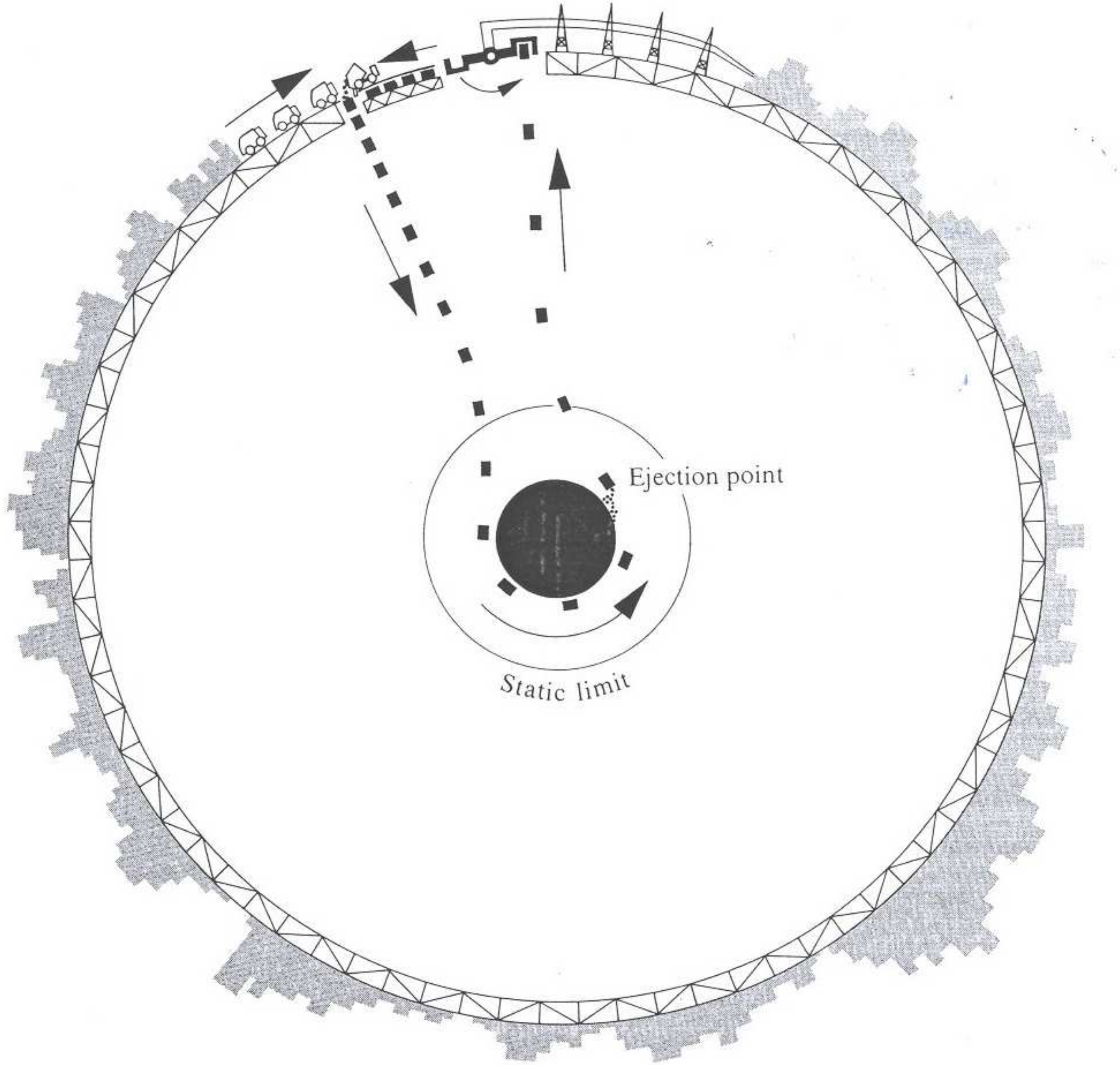
$$\Delta M = E_B = E_A - E_C < 0 \qquad \Delta J = L_B = L_A - L_C < 0$$



- $E_C > E_A$: Black hole delivers work on particle!
- $\Delta M < 0, \quad \Delta J < 0$: Black hole loses mass and angular momentum!

$\longrightarrow$ black holes are not just the sinks of the universe!

Misner, Thorme & Wheeler: Advanced society's recycling scheme



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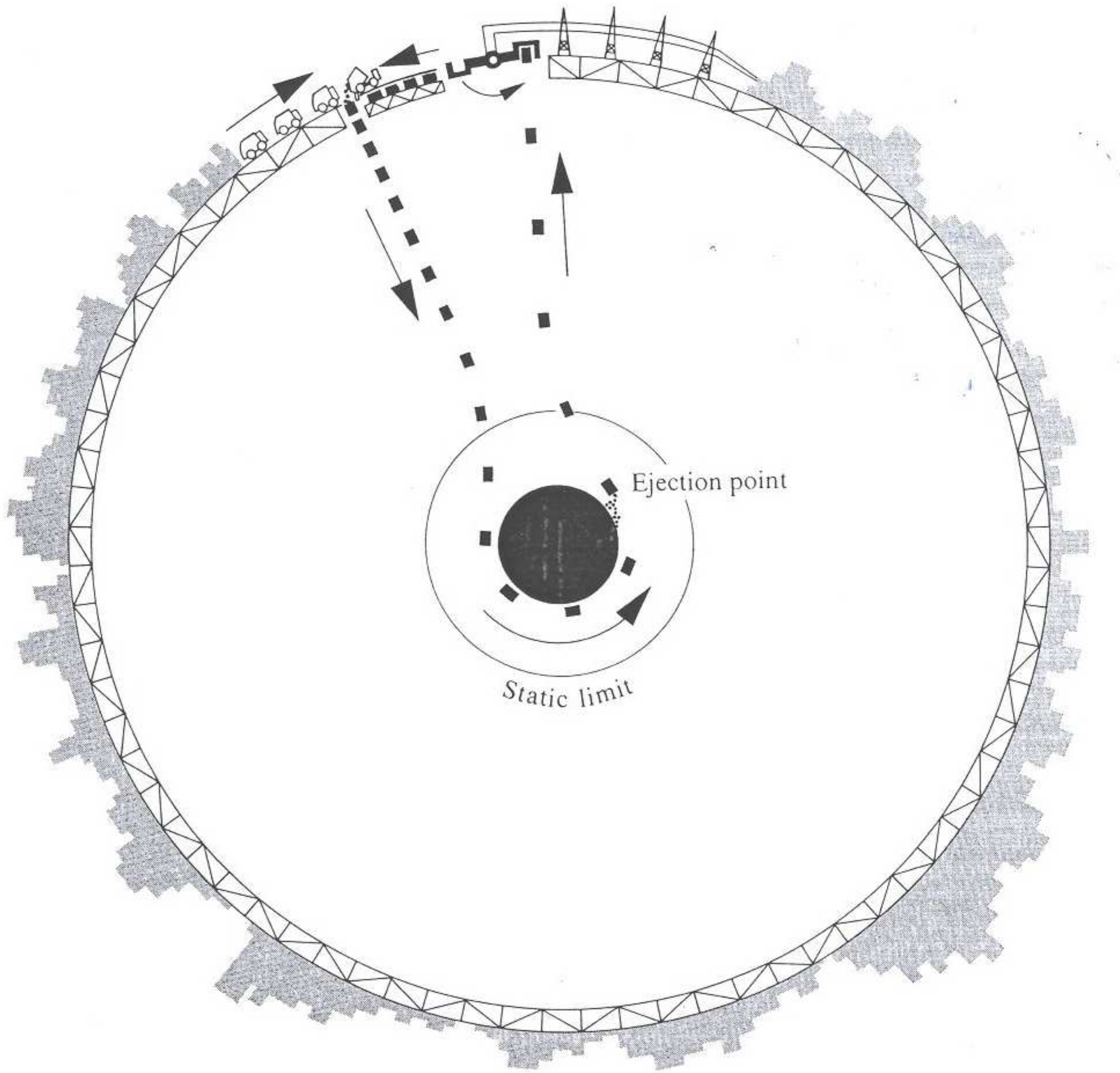

⟶ CT12: Sensibilidad hacia temas medioambientales

5. The area theorem

The area A of the horizon of a black hole never decreases in a physical process, not even in a Penrose process [Hawking, 1971]

$$A = 4\pi \left[2M^2 + 2M\sqrt{M^2 - a^2} \right] \qquad \text{(Kerr)}$$

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The irreducible mass M_* of a black hole never decreases in a physical proces, not even in a Penrose process [Christodoulou & Ruffini, 1971]

$$M^2 = M_*^2 + \frac{J^2}{4M_*^2} \qquad \text{(Kerr)}$$

with

$$M_*^2 = \tfrac{1}{4} \left[2M^2 + 2M\sqrt{M^2 - a^2} \right] \qquad \text{(Kerr)}$$

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In Penrose process: $\Delta M \leq \Omega_H \Delta J$

- non-optimal process: $\Delta M < \Omega_H \Delta J \quad \Longrightarrow \quad \Delta M_* > 0$ (irreversable)
- optimal process: $\Delta M = \Omega_H \Delta J \quad \Longrightarrow \quad \Delta M_* = 0$ (reversable)

So far...

- No-hair theorems: unique 3-parameter family of stationary black hole solutions, completely characterised by M, Q and J
- Penrose process: possible to extract (certain amount of) work from black holes $\Delta M \leq \Omega_H \Delta J$
- Area theorem: area $A = 16\pi M_*^2$ never descreases in physical processes, and only constant in reversable processes.

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$\longrightarrow$ smells like mechanical system!

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$\longrightarrow$ The analogy goes even further...

6. The laws of black hole mechanics [Bardeen, Carter, Hawking; 1973]

- **Zeroth law:** In stationary black holes, the surface gravity κ_H is constant along the horizon.

  NB: Surface gravity κ_H = acceleration of mass on horizon, measured by asymptotic observer
  = force aplied by asymptotic observer to maintain a mass stationary on horizon

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- **First law**: In quasi-stationary processes, M, A, J and Q vary like

$$dM = \frac{\kappa_H}{8\pi G_N} dA + \Omega_H dJ + \Phi_H dQ$$

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$$dM = \frac{\kappa_H}{8\pi G_N} dA + \Omega_H dJ + \Phi_H dQ$$

- **Second law**: The area A of the black hole horizon never descreases in physical processes.

- **Third law**: It is not possible to reduce the surface gravity κ_H to zero by physical processes in a finite time.

Compare with laws of thermodynamics: [Clausius; Kelvin; Gibbs; Nernst]

- **Zeroth law:** In systems in thermodynamic equilibrium, the temperature T is constant throughout the system.

- **First law**: In quasi-stationary processes, E, S, V and N vary like

$$dE = k_B T\, dS - P\, dV + \mu\, dN,$$

- **Second law**: The entropy of a closed system never descreases in physical processes.

- **Third law**: It is not possible to reduce the temperature T to zero by physical processes in a finite time.

Compare with laws of thermodynamics: [Clausius; Kelvin; Gibbs; Nernst]

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- **First law**: In quasi-stationary processes, E, S, V and N vary like

$$dE = k_B T\, dS - P\, dV + \mu\, dN,$$

- **Second law**: The entropy of a closed system never descreases in physical processes.
- **Third law**: It is not possible to reduce the temperature T to zero by physical processes in a finite time.

Suggests:

$M \sim E$	relativistic mass en energy
$\Omega_H dJ \sim -PdV$	work done by system
$A \sim S$	$\leftrightarrow$ uniqueness theorems: $S = 0$
$\kappa_H \sim T$	$\leftrightarrow$ black hole emits nothing: $T = 0$

7. Quantum black holes

Bardeen, Carter, Hawking: Pure analogy, no physical connection

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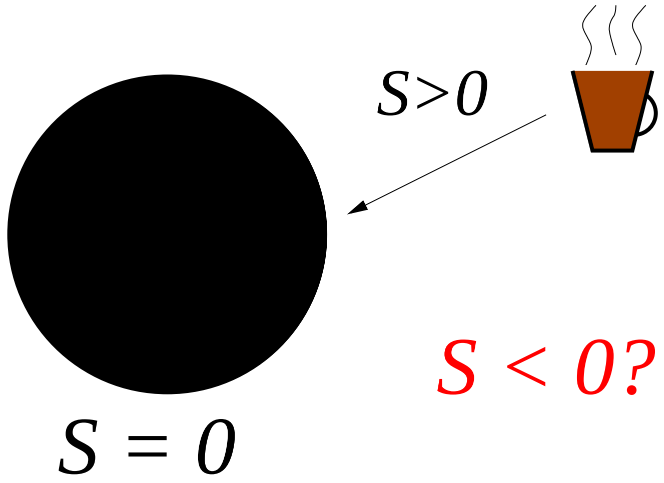

Black holes must have non-trivial entropy, in order to satisfy Second Law of Thermodynamics

$\longrightarrow$ Entropy is real and huge! $S \sim A/\ell_P^2$ [Bekenstein, 1973]

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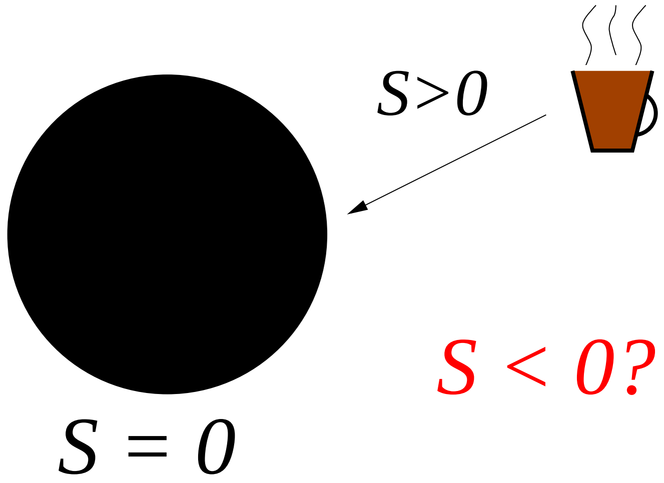

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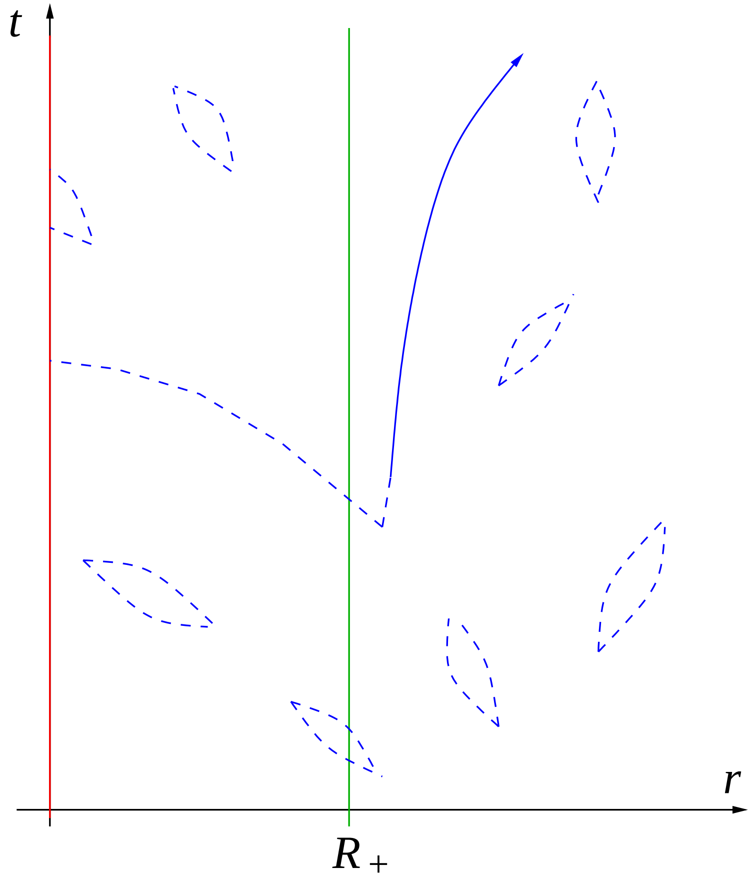
Question: How can black holes have temperature, if they do not emit anything?

$\longrightarrow$ Look at quantum character...

Hawking radiation

[Hawking, 1974]

QFT in curved spacetime: black hole behaves like black body



Schwinger pair creation near horizon

$\Longrightarrow$ thermal radiation with

$$T = \frac{\hbar\, \kappa_H}{2\pi k_B}$$

$\Longrightarrow$ identify entropy as

$$S = \frac{A}{4 G_N \hbar}$$

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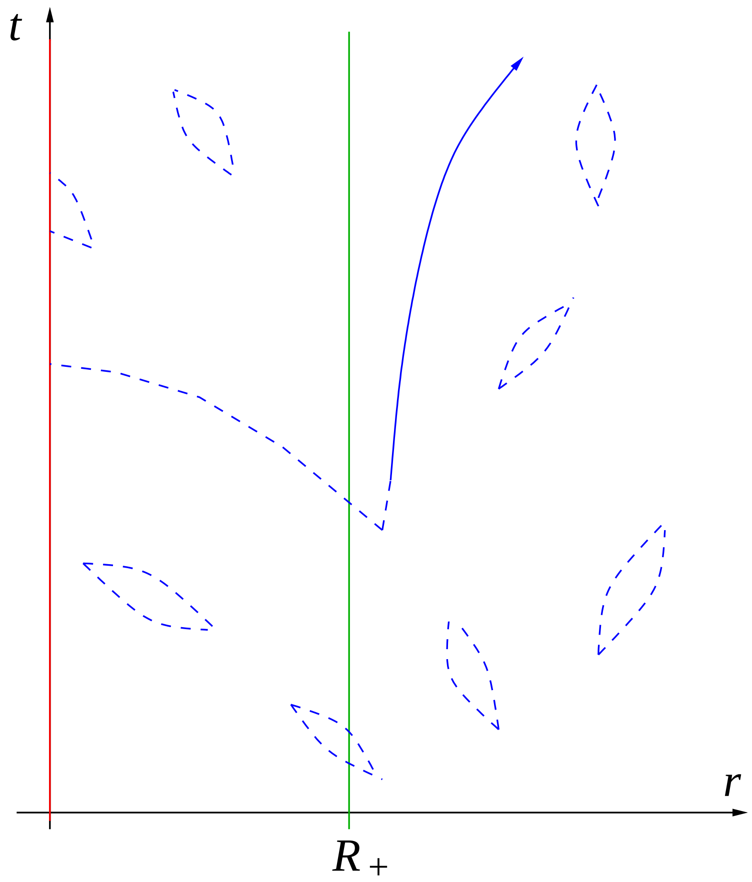

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Two big problems:

- Counting of microstates: what is nature of entropy?
- Information paradox: is time evolution unitary?

Counting of microstates

Boltzmann: $S = \ln N$ [Boltzmann, 1877]

Shannon: $S = \sum_i p_i \log_2 p_i$

$$M = m_\odot \quad \Longrightarrow \quad S \sim 10^{77}$$

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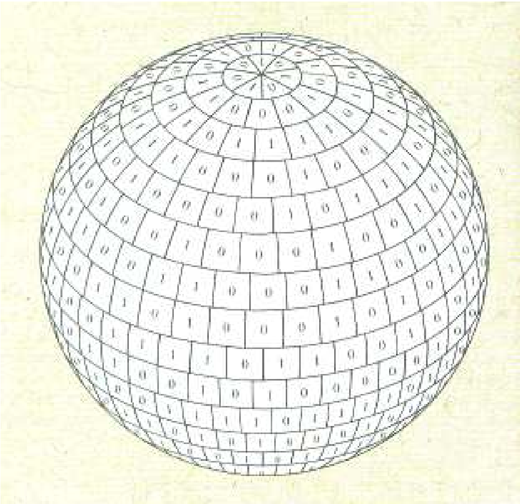

Degrees of freedom of gravitational system in volume V is described by quantum field theory on boundary ∂V

$\longrightarrow$ not just for black holes

(AdS/CFT, cosmology, ...)

- Entanglement entropy?

  State of Hawking pair:

$$|\psi_{\rm tot}\rangle \ = \ \tfrac{1}{\sqrt{2}}\Big(|0_{\rm int}\rangle \otimes |0_{\rm ext}\rangle \ + \ |1_{\rm int}\rangle \otimes |1_{\rm ext}\rangle\Big)$$

  Internal part inaccessable $\longrightarrow$ description in terms of density matrix

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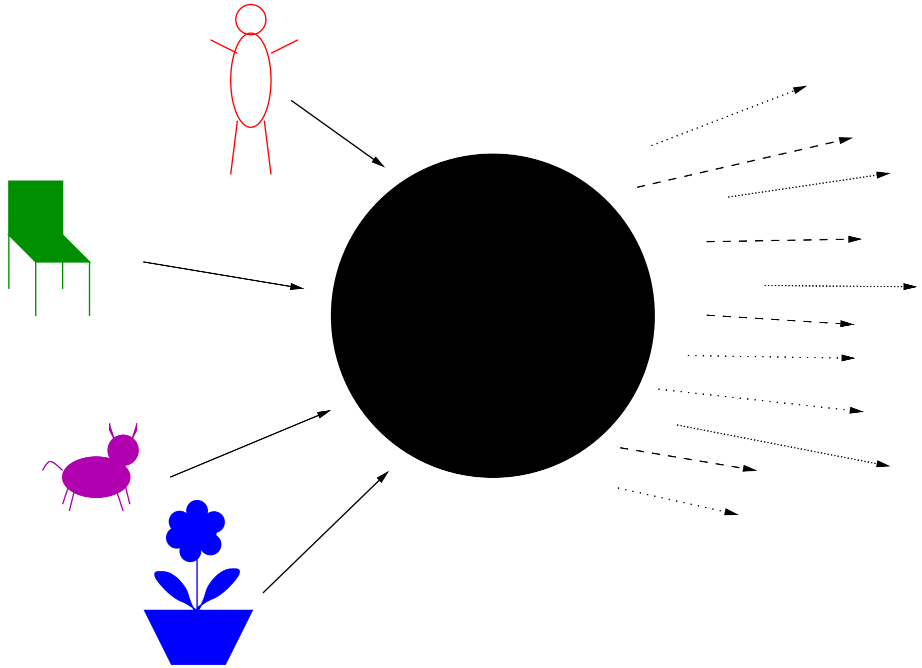
Information paradox

Where goes information about collapsing matter?



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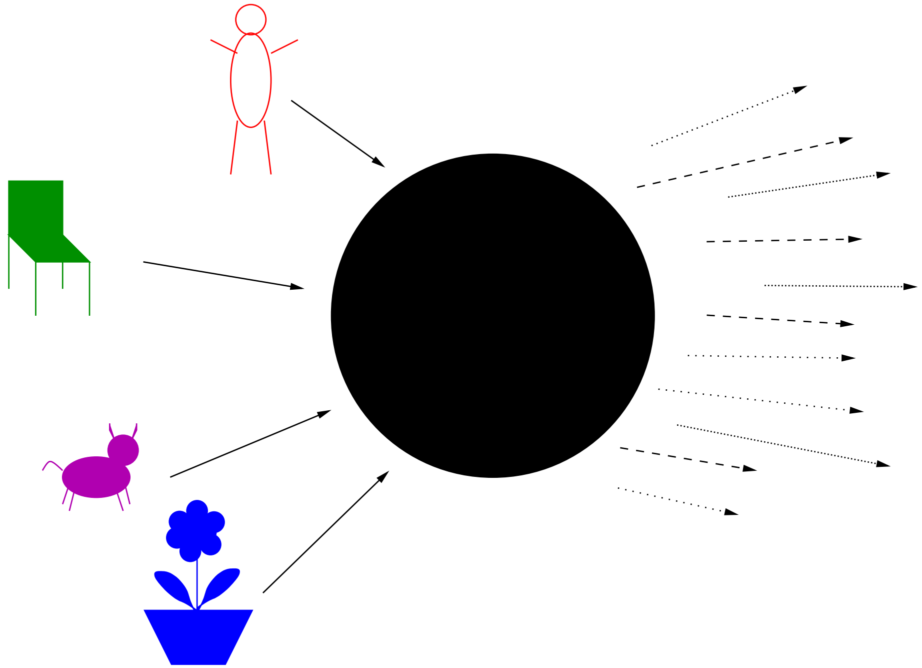

- Classically: inside de black hole, unaccessable...
- Quantum mechanically: black hole evaporation $\longrightarrow$ dispersion problem
  In-state = (sum of) pure states; Out-state = thermal state
  $\longrightarrow$ Violates unitarity of Quantum Mechanics!

Open Questions...

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Problem:

Incompatibility between Equivalence Principle, unitarity or locality?

$\longrightarrow$ which one shall we sacrifice?

Conclusions:

- General relativity is coarse-grained, (semi-)classical description of an underlying quantum system!

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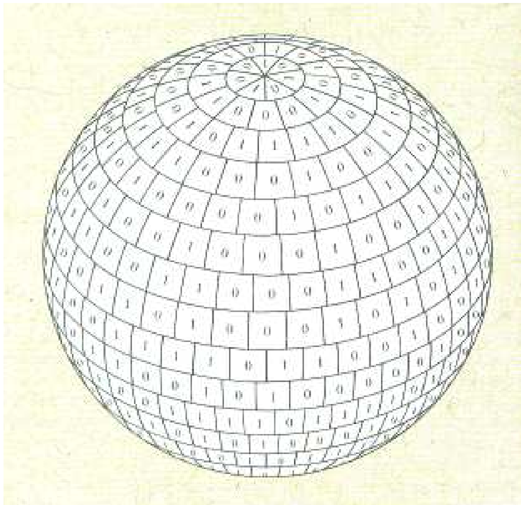
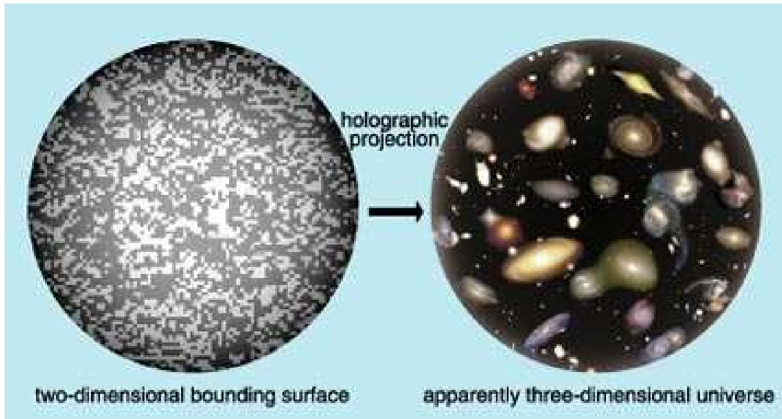

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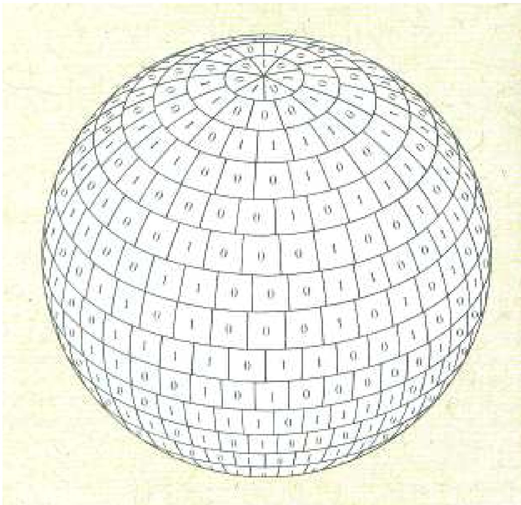
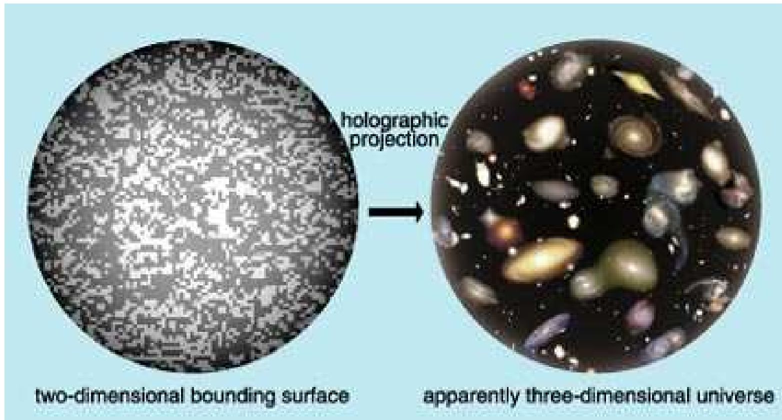
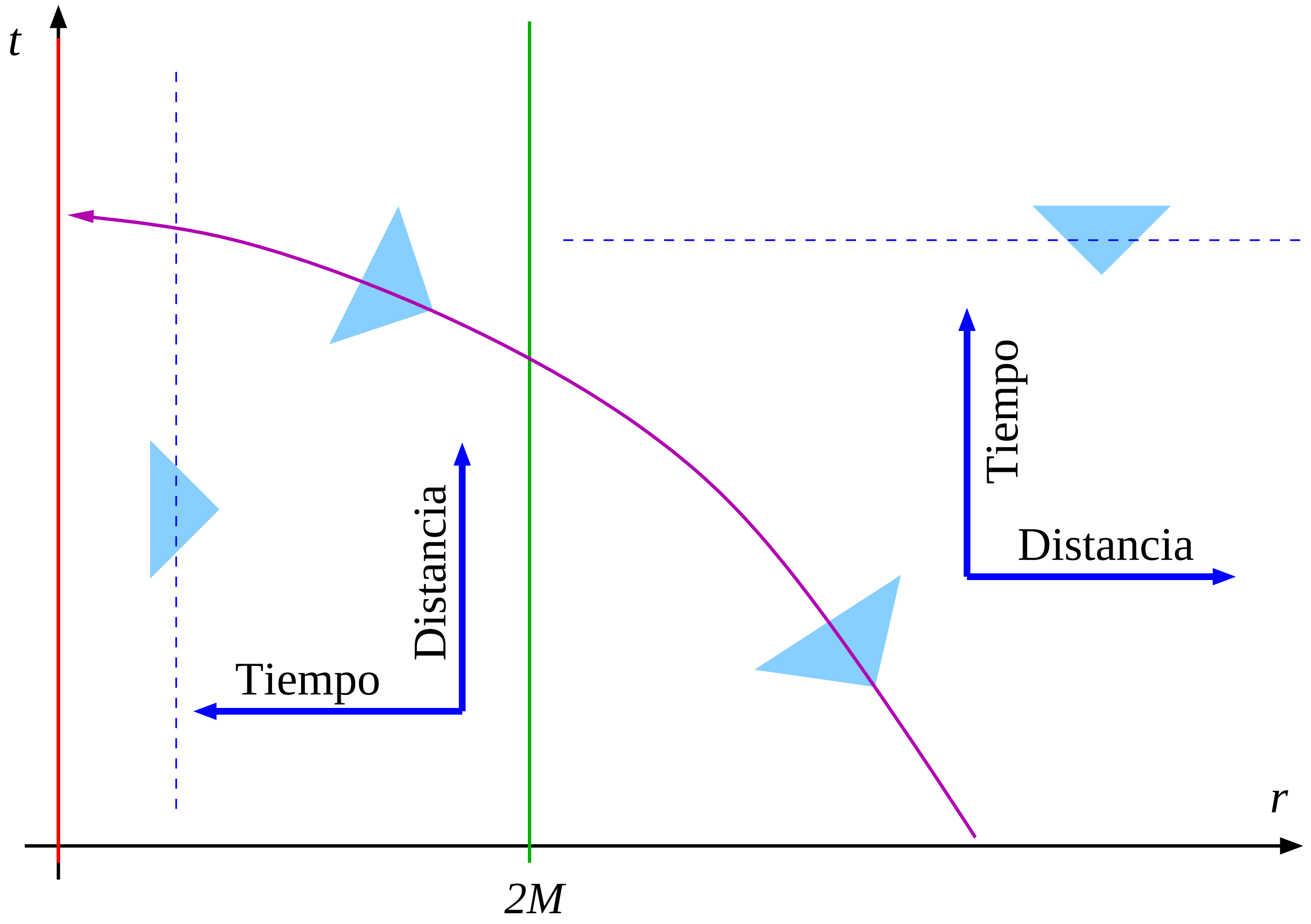



- Ultimate Question: What are degrees of freedom of quantum gravity?

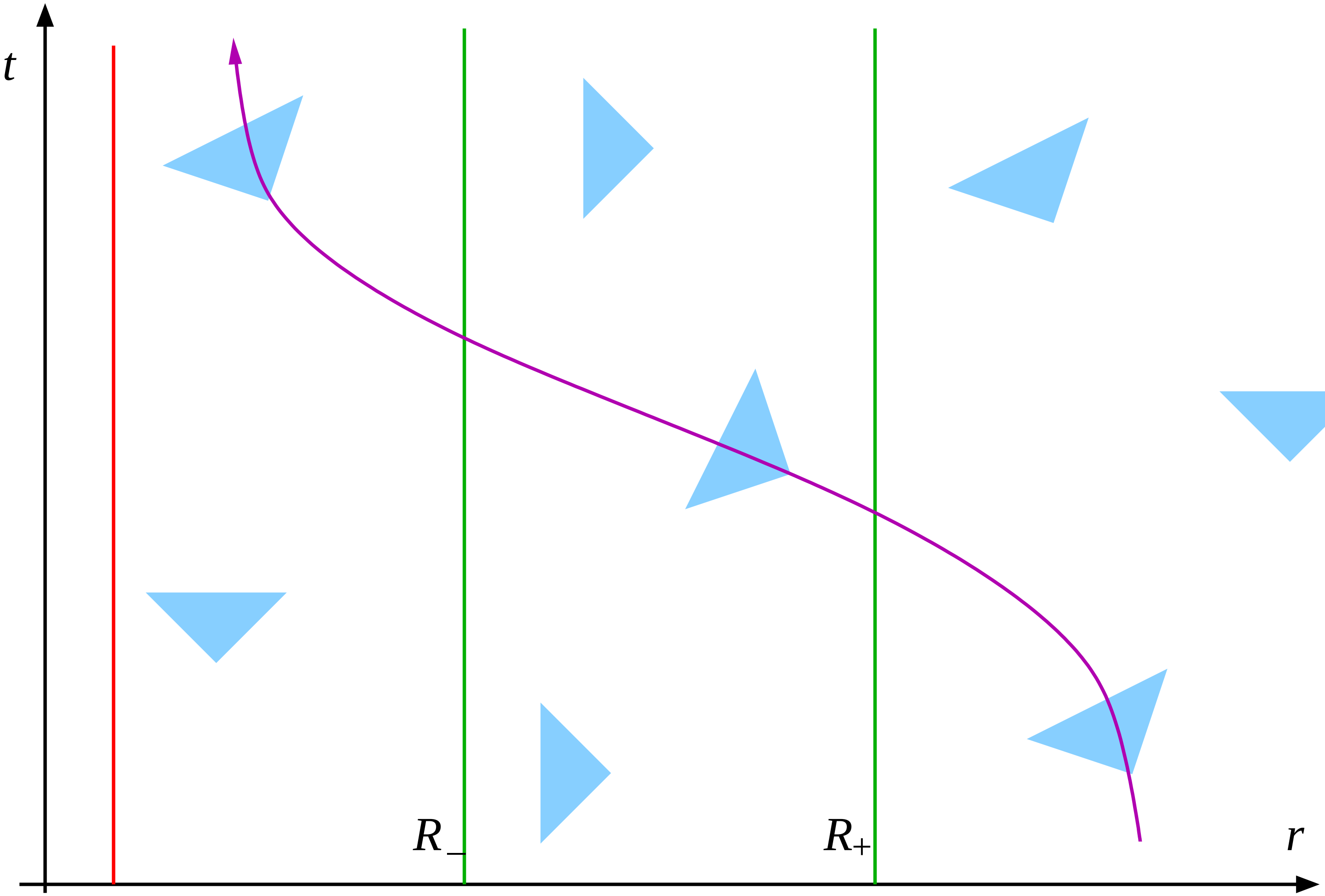
Thank you!

$g_{tt} = 1 - \frac{2M}{r}$ changes sign at $r = 2M$: t and r interchange roles



- $r > 2M$: t is timelike, r is spacelike: asymptotically flat region
- $r < 2M$: t is spacelike, r is timelike: non-static region

  $\longrightarrow$ $r = 0$ is spacelike singularity in future



- $R_+ < r$: t is timelike, r is spacelike: asymptotically flat region
- $R_- < r < R_+$: t is spacelike, r is timelike: non-stationary region
- $r < R_-$: t is timelike, r is spacelike: inner region

  $\longrightarrow$ ring singularity is timelike and localised

Analogue for Schwarzschild black hole:

Energy at infinity of particle at position r:

$$E(r) \;=\; g_{t\mu}\, p^{\mu} \;=\; m_0 \sqrt{1 - \frac{2M}{r}}$$

$$\longrightarrow \; E \;=\; m_0 \quad \text{at} \quad r = \infty$$

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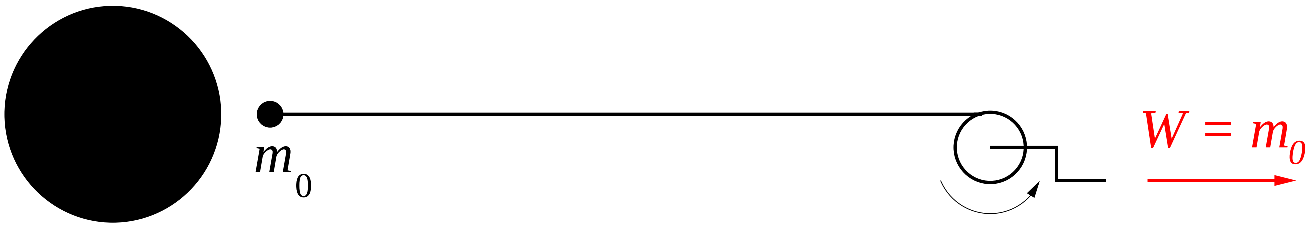


$\longrightarrow$ Possible to extract all energy of infalling particle and convert to work

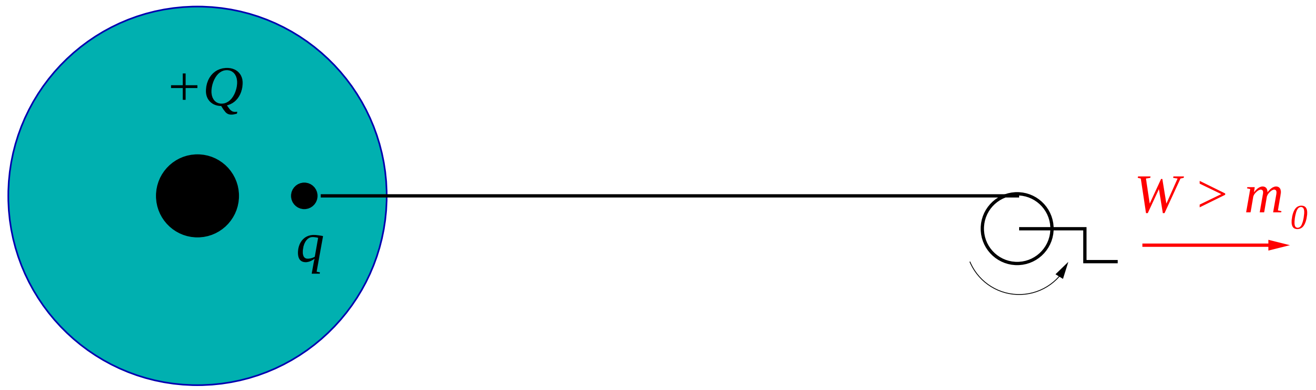
$$\Longrightarrow \quad \Delta M = 0$$

$\longrightarrow$ No energy extracted from black hole, but no energy added either

Analogue for charged black holes:

Canonical momentum of charged particle: $p^\mu = m_0 \, \dot{x}^\mu + qA^\mu$

$\longrightarrow E < 0$ if A^μ sufficiently negative



$\longrightarrow$ Possible to extract energy from black hole and convert to work

$$\Delta E = E(\infty) - E(R_+) = m_0 - \frac{qQ}{R_+} \qquad \text{(Reissner-Nordström)}$$

$\longrightarrow$ Extra work done by electromagnetic field of black hole...

... untill black hole is neutralised

Euclidean Path Integral

[Gibbons, Hawking, 1977]

QFT in euclidean space: periodic time $t = t + \beta$

finite temperature $T = 1/\beta$

Path integral:

$$Z(\beta) = \int \mathcal{D}\phi \, e^{-S_E[\phi]}$$

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$$Z(\beta) \sim \text{Tr } e^{-\beta H}$$

$\longrightarrow$ Interpret as opartition function in canonical ensemble:

$$E = M \qquad S = \frac{A}{4G_N\hbar} \qquad T = \frac{\hbar\,\kappa_H}{2\pi k_B}$$

$\longrightarrow$ Assuming thermodynamic description of some microscopic system:

Same results as geometrical approach!