Contact Holomorphic Curves and Flat Surfaces

J. A. Gálvez A. Martínez F. Milán*

Abstract

In this paper we study flat surfaces in the hyperbolic 3-space and the de Sitter 3-space with the conformal structure induced by its second fundamental form and give a conformal representation of such surfaces in terms of holomorphic data.

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1 Introduction

Partial differential equations on surfaces whose solutions could be represented in terms of holomorphic functions on Riemann surfaces have been extensively investigated. Famous examples are Laplace's equation $\Delta u = 0$ and Liouville's equation $\Delta u = e^u$.

An example from geometry is the minimal surface equation in the Euclidean space $\mathbb{R}^3$ whose holomorphic representation gives the global version of the Enneper-Riemann-Weierstrass representation, which is essentially due to Osserman [O]. This representation has been crucial in both reaching a rather exhaustive understanding and finding examples of complete minimal surfaces. In spaces of other constant sectional curvature such as the hyperbolic 3-space $\mathbb{H}^3$ or the de Sitter 3-space $\mathbb{S}^3_1$ the equation of a surface of constant mean curvature admits a holomorphic resolution that provides a global complex representation which has been used in the study of global properties of these surfaces, (see [AA], [B], [UY]).

The fully non-linear Monge-Ampère equation $\det \nabla^2 u = 1$ which arises in affine differential geometry (see [FMM], [J]) and in the study of the second fundamental form of flat surfaces in $\mathbb{H}^3$ and $\mathbb{S}^3_1$, can be solved using holomorphic data. In this paper we consider flat surfaces in $\mathbb{H}^3$ and $\mathbb{S}^3_1$ with the conformal structure induced by its second fundamental form. We will prove that these surfaces share a fundamental property with minimal surfaces in $\mathbb{R}^3$

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and surfaces of constant mean curvature in $\mathbb{H}^3$ and $\mathbb{S}^3_1$, they possess a "conformal representation" in terms of holomorphic data which involve its "hyperbolic" Gauss map (Theorem 1).

2 Some Preliminaries

Let $\mathbb{L}^4$ be the Minkowski 4-space endowed with linear coordinates (x_0, x_1, x_2, x_3) and the scalar product, $\langle .,. \rangle$ given by the quadratic form $-x_0^2 + x_1^2 + x_2^2 + x_3^2$. We set the two hyperquadrics

$$\mathbb{H}^3 = \left\{ (x_0, x_1, x_2, x_3) \in \mathbb{L}^4 / \; -x_0^2 + x_1^2 + x_2^2 + x_3^2 = -1,\, x_0 > 0 \right\},$$

$$\mathbb{S}^3_1 = \left\{ (x_0, x_1, x_2, x_3) \in \mathbb{L}^4 / \; -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 1 \right\},$$

with the induced metric from $\mathbb{L}^4$. Then, $\mathbb{H}^3$ is a Riemannian 3-manifold of constant sectional curvature -1 which is called the hyperbolic 3-space. $\mathbb{S}^3_1$ is a 3-dimensional Lorentzian manifold of constant sectional curvature 1 and it is called the de Sitter 3-space.

Let $\mathbb{N}^3$ denote the positive null cone, that is

$$\mathbb{N}^3 = \left\{ (x_0, x_1, x_2, x_3) \in \mathbb{L}^4 / \; -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0,\, x_0 > 0 \right\}.$$

If one considers for all $v \in \mathbb{N}^3$ the halfline $[v]$ spanned by v, then this gives a partition of $\mathbb{N}^3$ and the ideal boundary $\mathbb{S}^2_\infty$ of $\mathbb{H}^3$ can be regarded as the quotient of $\mathbb{N}^3$ under the associated equivalence relation. Thus, the induced metric is well-defined up to a factor and $\mathbb{S}^2_\infty$ inherits a natural conformal structure as the quotient $\mathbb{N}^3/\mathbb{R}^+$.

We consider $\mathbb{L}^4$ identified with the space of 2×2 Hermitian matrices, Herm(2), by identifying $(x_0, x_1, x_2, x_3) \in \mathbb{L}^4$ with the matrix

$$\begin{pmatrix} x_0 + x_3 & x_1 + ix_2 \\ x_1 - ix_2 & x_0 - x_3 \end{pmatrix} = \sum_{j=0}^{3} x_j e_j, \tag{1}$$

where

$$e_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad e_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Under this identification, one has $\langle m, m \rangle = -\det(m)$, for all $m \in$ Herm(2), and the complex Lie group $\mathbf{SL}(2, \mathbb{C})$ of 2×2 complex matrices with determinant 1 acts naturally on $\mathbb{L}^4$ by the representation

$$g \cdot m = gmg^*,$$

where $g \in \mathbf{SL}(2, \mathbb{C})$, $g^* = {}^t\overline{g}$ and $m \in$ Herm(2). Consequently, $\mathbf{SL}(2, \mathbb{C})$ preserves the scalar product and orientations. The kernel of this action is $\{\pm I_2\} \subseteq \mathbf{SL}(2, \mathbb{C})$ and $\mathbf{PSL}(2, \mathbb{C}) = \mathbf{SL}(2, \mathbb{C}) / \{\pm I_2\}$ can be regarded as the identity component of the special Lorentzian group $\mathbf{SO}(1, 3)$. This action can be restricted to $\mathbb{H}^3$ and $\mathbb{S}^3_1$ as an isometric and transitive one. Thus, $\mathbb{H}^3$ and $\mathbb{S}^3_1$ can also be represented as

$$\mathbb{H}^3 = \{ g \cdot e_0 / \; g \in \mathbf{SL}(2, \mathbb{C}) \}$$

and

$$\mathbb{S}^3_1 = \{g \cdot e_j /\ g \in \ \mathbf{SL}(2,\mathbb{C})\}, \quad j \in \{1,2,3\}.$$

The space $\mathbb{N}^3$ is seen as the space of positive semi-definite 2×2 Hermitian matrices of determinant 0 and its elements can be written as $a\,{}^t\overline{a}$, where ${}^t a = (a_1, a_2)$ is a non-zero vector in $\mathbb{C}^2$ uniquely defined up to multiplication by an unimodular complex number. The map $a\,{}^t\overline{a} \longrightarrow [(a_1,a_2)] \in \mathbb{C}\mathbf{P}^1$ becomes the quotient map of $\mathbb{N}^3$ on $\mathbb{S}^2_\infty$ and identifies $\mathbb{S}^2_\infty$ with $\mathbb{C}\mathbf{P}^1$. So the natural action of $\mathbf{SL}(2,\mathbb{C})$ on $\mathbb{S}^2_\infty$ is the action of $\mathbf{SL}(2,\mathbb{C})$ on $\mathbb{C}\mathbf{P}^1$ by Möbius transformations.

3 Contact Holomorphic Curves

On

$$\mathbf{SL}(2,\mathbb{C}) = \left\{ \underline{z} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \mid \ \det(\underline{z}) = 1 \right\}$$

we shall consider the canonical contact structure induced by the contact 1-form

$$\Omega \equiv z_{22} dz_{11} - z_{12} dz_{21}.$$

Let Σ be a Riemann surface and $g : \Sigma \longrightarrow \mathbf{SL}(2,\mathbb{C})$,

$$g = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}, \qquad G_{11}G_{22} - G_{12}G_{21} = 1,$$

be a holomorphic map such that $g^*\Omega$ vanishes on Σ, then

$$g^{-1}dg = \begin{pmatrix} G_{22} & -G_{12} \\ -G_{21} & G_{11} \end{pmatrix} \begin{pmatrix} dG_{11} & dG_{12} \\ dG_{21} & dG_{22} \end{pmatrix} = \begin{pmatrix} 0 & \alpha_{12} \\ \alpha_{21} & 0 \end{pmatrix}.$$

Definition 1 *A holomorphic map $g : \Sigma \longrightarrow \mathbf{SL}(2,\mathbb{C})$ is called a contact holomorphic map if $g^*\Omega \equiv 0$ and α_{21} never vanishes on Σ.*

Thus, if we set

$$f = \frac{\alpha_{12}}{\alpha_{21}}, \qquad \omega = \alpha_{21},$$

the pair (f, ω) satisfies the following equality

$$(2) \qquad g^{-1}dg = \begin{pmatrix} 0 & f \\ 1 & 0 \end{pmatrix} \omega.$$

Conversely, let f be a holomorphic function and ω a holomorphic 1-form on Σ such that $\omega \neq 0$ everywhere. Then the ordinary differential equation (2) is integrable and a solution g is a contact holomorphic map into $\mathbf{SL}(2,\mathbb{C})$ but g may not be well-defined on Σ. In fact, when we consider (f,ω) written in an arbitrary complex parameter ζ as $(f(\zeta), h(\zeta)d\zeta)$, every solution g of (2) is given as

$$(3) \qquad g = \begin{pmatrix} V & \frac{1}{h} V_\zeta \\ W & \frac{1}{h} W_\zeta \end{pmatrix}$$

where V and W are linearly independent solutions of the ordinary linear differential equation

$$X_{\zeta\zeta} - \frac{h_\zeta}{h} X_\zeta - f h^2 X \ = \ 0, \tag{4}$$

and

$$h = V W_\zeta - W V_\zeta.$$

Definition 2 *The pair (f, ω) will be called Weierstrass data (complex representation) of g.*

4 A Conformal Representation

Theorem A) *Let Σ be a Riemann surface and $g : \Sigma \longrightarrow \mathbf{SL}(2, \mathbb{C})$ a contact holomorphic map with Weierstrass data (f, ω).*

I) *If $|f| < 1$, then $\psi_0 = g \cdot e_0 : \Sigma \longrightarrow \mathbb{H}^3$ and $\psi_3 = g \cdot e_3 : \Sigma \longrightarrow \mathbb{S}^3_1$ are well-defined flat Riemannian immersions.*

II) *If the imaginary part of f, $\Im(f)$, never vanishes on Σ, then $\psi_1 = g \cdot e_1 : \Sigma \longrightarrow \mathbb{S}^3_1$ is a well-defined flat Lorentzian immersion.*

B) *Conversely, let M be a simply connected surface and $\psi : M \longrightarrow N$ a flat immersion, where N is either $\mathbb{H}^3$ or $\mathbb{S}^3_1$. If on M we consider the conformal structure determined by the second fundamental form of ψ, then there exists a contact holomorphic map $g : M \longrightarrow \mathbf{SL}(2, \mathbb{C})$ with Weierstrass data (f, ω) such that $\psi = \psi_j$ for some $j \in \{0, 1, 3\}$. Moreover g is unique up to right multiplication by a constant matrix g_0 with $g_0 \cdot e_j = e_j$.*

Proof: We consider $\psi = g \cdot e$ with

$$e = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \end{pmatrix}, \qquad \epsilon = \pm 1.$$

Then from (2),

$$d\psi = g \begin{pmatrix} 0 & \epsilon f \omega + \overline{\omega} \\ \omega + \epsilon \overline{f} \overline{\omega} & 0 \end{pmatrix} g^*,$$

and the induced metric is given by

$$ds^2 = \langle d\psi, d\psi \rangle = -\det(d\psi) = (1 + f\overline{f})\omega\overline{\omega} + \epsilon f \omega^2 + \epsilon \overline{f} \overline{\omega}^2.$$

Since f is a holomorphic function

$$d\left(\frac{1}{2}(1 + \epsilon f)\omega + \frac{1}{2}(1 + \epsilon \overline{f})\overline{\omega}\right) = 0, \qquad d\left(\frac{i}{2}(1 - \epsilon f)\omega - \frac{i}{2}(1 - \epsilon \overline{f})\overline{\omega}\right) = 0$$

and there exist local functions x, y such that

$$dx = \frac{1}{2}(1 + \epsilon f)\omega + \frac{1}{2}(1 + \epsilon \overline{f})\overline{\omega}, \qquad dy = \frac{i}{2}(1 - \epsilon f)\omega - \frac{i}{2}(1 - \epsilon \overline{f})\overline{\omega}.$$

with

$$dx \wedge dy = \frac{-i}{2}(1 - |f|^2)\omega \wedge \overline{\omega}.$$

Because of $|f| < 1$, (x, y) are new coordinates with $ds^2 = dx^2 + dy^2$. Therefore, the immersion is flat.

In the same way, when $\psi = \psi_1$ one gets $ds^2 = 2dxdy$, where

$$dx = -f\omega - \overline{f}\overline{\omega}, \qquad dy = \frac{1}{2}\omega + \frac{1}{2}\overline{\omega}.$$

Conversely, if $\psi : M \longrightarrow \mathbb{S}^3_1$ is a Lorentzian immersion with flat induced metric, then there exists an asymptotic coordinate immersion $x + iy : M \longrightarrow \mathbb{C}$ such that

$$ds^2 = 2dxdy, \tag{5}$$

and if η is an unit normal vector field to the immersion, a straight calculation gives the following structure equations

$$\begin{aligned} \psi_{xx} &= E\eta, \\ \psi_{xy} &= F\eta - \psi, \\ \psi_{yy} &= G\eta, \\ \eta_x &= -F\psi_x - E\psi_y, \\ \eta_y &= -G\psi_x - F\psi_y, \end{aligned} \tag{6}$$

where E, F and G are smooth functions on M and by $(\cdot)_x$ and $(\cdot)_y$ we shall denote the usual partial derivatives respect to x and y, respectively.

Using the Gauss' and Codazzi-Mainardi's equations we have $EG - F^2 = 1$, $E_y = F_x$ and $F_y = G_x$. Hence, as M is simply connected, there exists a well-defined function ϕ on M such that $E = \phi_{xx}$, $F = \phi_{xy}$, $G = \phi_{yy}$ and the second fundamental form of the immersion is given by

$$d\sigma^2 = \phi_{xx}dx^2 + \phi_{yy}dy^2 + 2\phi_{xy}dxdy, \tag{7}$$

with

$$\phi_{xx}\phi_{yy} - \phi_{xy}^2 = 1. \tag{8}$$

We shall regard M as a Riemann surface with the conformal structure determined by the second fundamental form $d\sigma^2$.

From (8), we can choose η such that $\phi_{xx} > 0$ and consider the new coordinate immersion

$$z = u + iv = y - i\phi_x. \tag{9}$$

Then, a straight computation gives,

$$\psi_u = -\frac{\phi_{xy}}{\phi_{xx}}\psi_x + \psi_y, \qquad \psi_v = \frac{-1}{\phi_{xx}}\psi_x. \tag{10}$$

Now, from (6), (7), (8), (9) and (10) we have

$$d\sigma^2 = \frac{1}{\phi_{xx}}|dz|^2 \tag{11}$$

and

$$(\psi_x)_u = -\psi, \qquad (\psi_x)_v = -\eta. \tag{12}$$

Thus, from (5), (9), (11) and (12), $z : M \longrightarrow \mathbb{C}$ is a conformal coordinate immersion and $[\psi_x] : M \longrightarrow \mathbb{S}^2_\infty$ is a conformal map, which induces on M the flat Riemannian metric $|dz|^2$.

Moreover, from the above expressions, we obtain

$$(\psi_x)_{uu} = \frac{\phi_{xy}}{\phi_{xx}}\psi_x - \psi_y, \qquad (\psi_x)_{vv} = -\frac{\phi_{xy}}{\phi_{xx}}\psi_x - \psi_y,$$

and by using standard notations of complex analysis, one has

$$4(\psi_x)_{z\overline{z}} = -2\psi_y. \tag{13}$$

Now, let $A, B : M \longrightarrow \mathbb{C}$ be global holomorphic functions on M such that $[\psi_x]$ is represented as $[(A, B)] \in \mathbb{C}\mathbf{P}^1 \equiv \mathbb{S}^2_\infty$, then

$$\psi_x = \lambda \begin{pmatrix} A \\ B \end{pmatrix} \left(\overline{A}, \overline{B}\right) = \lambda \begin{pmatrix} A\overline{A} & A\overline{B} \\ \overline{A}B & B\overline{B} \end{pmatrix},$$

for some positive function $\lambda \in C^\infty(M)$. Thus, from (5), (9) and (12), one gets

$$\frac{1}{2} = \langle (\psi_x)_z, (\psi_x)_{\overline{z}} \rangle = \frac{1}{2}\lambda^2 \left| AB_z - BA_z \right|^2$$

and as $AB_z - BA_z$ does not vanish on the simply connected surface M, there exists a holomorphic function $R : M \longrightarrow \mathbb{C}$ with $R^2 = AB_z - BA_z$. Hence, we can write

$$\psi_x = \begin{pmatrix} C\overline{C} & C\overline{D} \\ \overline{C}D & D\overline{D} \end{pmatrix}, \tag{14}$$

where $C = A/R$ and $D = B/R$. Consequently, from (12) and (14) we have the following expression for the immersion

$$\psi = -\begin{pmatrix} C\overline{C_z} + C_z\overline{C} & C\overline{D_z} + C_z\overline{D} \\ \overline{C}D_z + \overline{C_z}D & D\overline{D_z} + D_z\overline{D} \end{pmatrix} \tag{15}$$

and for its unit normal

$$\eta = -i \begin{pmatrix} -C\overline{C_z} + C_z\overline{C} & -C\overline{D_z} + C_z\overline{D} \\ \overline{C}D_z - \overline{C_z}D & -D\overline{D_z} + D_z\overline{D} \end{pmatrix}.$$

If we consider the function $f : M \longrightarrow \mathbb{C}$ defined by

$$f = \frac{\phi_{xy} - i}{2\phi_{xx}}, \tag{16}$$

then, from (6), (10), (12), (13) and (14)we obtain $(\psi_y)_z = -2f(\psi_x)_{\overline{z}}$ and

$$\begin{pmatrix} C_{zz}\overline{C_z} & C_{zz}\overline{D_z} \\ \overline{C_z}D_{zz} & D_{zz}\overline{D_z} \end{pmatrix} = f \begin{pmatrix} C\overline{C_z} & C\overline{D_z} \\ \overline{C_z}D & D\overline{D_z} \end{pmatrix}.$$

As C_z and D_z cannot vanishing simultaneously, we have

$$C_{zz} = fC, \qquad D_{zz} = fD. \tag{17}$$

Thus, from (8), (16) and (17), f is a holomorphic function which satisfies

$$\Im(f) \neq 0.$$

Finally, from (15) and (17), the immersion ψ can be recovered as $\psi = -g \cdot e_1$, where $g : M \longrightarrow \mathbf{SL}(2, \mathbb{C})$ is the contact holomorphic map given by

$$g = \begin{pmatrix} C & C_z \\ D & D_z \end{pmatrix} \tag{18}$$

such that

$$g^{-1} dg = \begin{pmatrix} 0 & f \\ 1 & 0 \end{pmatrix} \omega \tag{19}$$

and $\omega = dz$. On the other hand, if $\widetilde{g} : M \longrightarrow \mathbf{SL}(2, \mathbb{C})$ is a holomorphic immersion with $\psi = -\widetilde{g} \cdot e_1$, then there exists a holomorphic map $g_0 : M \longrightarrow \mathbf{SL}(2, \mathbb{C})$ such that $g = \widetilde{g} g_0$, with $g_0 \cdot e_1 = e_1$. Thus $(g_0)_z = 0$ and g_0 must be constant.

Now, let $\psi : M \longrightarrow N$ be a Riemannian immersion with flat induced metric, where $N = \mathbb{H}^3$ or $N = \mathbb{S}^3_1$, then there exists a coordinate immersion $x + iy : M \longrightarrow \mathbb{C}$ such that

$$ds^2 = dx^2 + dy^2. \tag{20}$$

The structure equations are given by

$$\begin{aligned} \psi_{xx} &= E\eta + \epsilon\psi, \\ \psi_{xy} &= F\eta, \\ \psi_{yy} &= G\eta + \epsilon\psi, \\ \eta_x &= -\epsilon E \psi_x - \epsilon F \psi_y, \\ \eta_y &= -\epsilon F \psi_x - \epsilon G \psi_y, \end{aligned}$$

where $\epsilon = 1$ if $N = \mathbb{H}^3$, $\epsilon = -1$ if $N = \mathbb{S}^3_1$, E, F and G are smooth functions on M and η is the well-oriented unit normal vector field. From the integrability conditions, there exists a well-defined function ϕ on M such that $E = \phi_{xx}$, $F = \phi_{xy}$, $G = \phi_{yy}$ satisfying (8) and the second fundamental form of the immersion is given by (7).

We consider the new coordinate immersion

$$z = u + iv = x + \phi_x + i(y + \phi_y).$$

Then

$$d\sigma^2 = \frac{1}{2 + \phi_{xx} + \phi_{yy}} |dz|^2$$

and

$$(\psi - \eta)_u = \psi_x, \qquad (\psi - \eta)_v = \psi_y.$$

Thus, $z : M \longrightarrow \mathbb{C}$ is a conformal coordinate immersion and $[\psi - \eta] : M \longrightarrow \mathbb{S}^2_\infty$ is a conformal map.

Calculating $(\psi - \eta)_{uu}$, $(\psi - \eta)_{vv}$, we obtain

$$\psi = \frac{1}{2}(\psi - \eta) + 2\epsilon(\psi - \eta)_{z\bar{z}}.$$

As in the above case, (see[GMM]), the immersion can be calculated as

$$\psi = \begin{pmatrix} C\overline{C} + 4\epsilon C_z\overline{C_z} & C\overline{D} + 4\epsilon C_z\overline{D_z} \\ \overline{C}D + 4\epsilon\overline{C_z}D_z & D\overline{D} + 4\epsilon D_z\overline{D_z} \end{pmatrix}$$

where C and D are linearly independent solutions of the ordinary linear differential equation

$$X_{zz} = \frac{1}{4}fX,$$

and $f : M \longrightarrow \mathbb{C}$ is the holomorphic function defined by

$$f = \frac{\phi_{yy} - \phi_{xx} + 2i\phi_{xy}}{2 + \phi_{xx} + \phi_{yy}}.$$

Moreover, from (8), $|f| < 1$.

Finally, the immersion ψ can be recovered as $\psi = g \cdot e_0$ or $\psi = g \cdot e_3$ if $N = \mathbb{H}^3$ or $N = \mathbb{S}^3_1$, respectively, where $g : M \longrightarrow \mathbf{SL}(2, \mathbb{C})$ is the contact holomorphic map given as in (18) satisfying (19) and $\omega = \frac{1}{2}dz$. □

Remark *The above conformal representation can be used in the study of global properties of flat surfaces in* $\mathbb{H}^3$ *and* $\mathbb{S}^3_1$*. For the particular case of flat surfaces in* $\mathbb{H}^3$ *the reader can see* [GMM].

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José A. Gálvez, Antonio Martínez and Francisco Milán
Departamento de Geometría y Topología
Facultad de Ciencias
Universidad de Granada
18071 GRANADA. SPAIN
(e-MAIL: jagalvez@goliat.ugr.es; amartine@goliat.ugr.es; milan@goliat.ugr.es)

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