We study holomorphic immersions of open Riemann surfaces into $\mathbb{C}^n$ whose derivative lies in a conical algebraic subvariety A of $\mathbb{C}^n$ that is smooth away from the origin. Classical examples of such A-immersions include null curves in $\mathbb{C}^3$ which are closely related to minimal surfaces in $\mathbb{R}^3$, and null curves in $SL_2(\mathbb{C})$ that are related to Bryant surfaces. We establish a basic structure theorem for the set of all A-immersions of a bordered Riemann surface, and we prove several approximation and desingularization theorems. Assuming that A is irreducible and is not contained in any hyperplane, we show that every A-immersion can be approximated by A-embeddings; this holds in particular for null curves in $\mathbb{C}^3$. If in addition $A \setminus \{0\}$ is an Oka manifold, then A-immersions are shown to satisfy the Oka principle, including the Runge and the Mergelyan approximation theorems. Another version of the Oka principle holds when A admits a smooth Oka hyperplane section. This lets us prove in particular that every open Riemann surface is biholomorphic to a properly embedded null curve in $\mathbb{C}^3$.